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Response of Fabry–Perot interferometers to light pulses of very short duration

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Characteristic fringes of a spectrometer are produced by interference of a large number of wavefronts of regularly increasing phase difference. This phase difference implies a temporal delay between the wavefronts. So, the response of such an instrument to a light pulse of very short duration may not be given by the conventional formula, which generally corresponds to a steady-state situation. In this paper, the temporal response of Fabry–Perot interferometers to pulses of light of various lengths and kinds is considered and inferences are drawn regarding how to estimate the pulse lengths and how to carry out the spectral analysis of such pulses.

Index Headings: Interferometers; Laser.

Spectrometers like the Fabry–Perot interferometer and gratings give rise to fringes by interference between a large number of beams with a regular phase difference between the consecutive beams produced by the spectrometer. This regular phase (path) difference implies a regular delay time (τ_0) between consecutive beams. Thus, a spectrometer would always need a finite characteristic response time ($\tau_r = p\tau_0$) after the incidence of the parent wavefront in which to build up the interference of multiple beams to produce the characteristic fringes (where p is the effective number of interfering beams). Further, such characteristic fringes can be formed only if the series of beams produced by the spectrometer is simultaneously present at the location or time of interest. This implies that the incident parent wave train must have a temporal duration, $\delta t \geq \tau_r$, to produce such characteristic fringes. This dynamic condition is generally met by the well-known static criterion that the free spectral range of the spectrometer should be larger than the width of the spectrum to be analyzed (this point will be further clarified in the following sections). Thus, it is important to realize that a spectrometer of response time τ_r will not show a normal spectral response if the incident wave train has a temporal duration less than τ_r. This point should be borne in mind, especially, in spectroscopy of pico- and nanosecond light pulses, where τ_r may be of this order or larger.

In this paper, we attempt an analysis of such situations, including some interesting cases when the incident wavefront consists of a pulse or a train of regular pulses of durations comparable to the spectrometer delay time τ_0. The Fabry–Perot interferometer being the most flexible and representative of the spectrometers, we shall do all the analysis with reference to this interferometer. The next section is a brief review of the Fabry–Perot interferometer (FP) in order to define terms that we shall use. In the subsequent sections, we shall derive the response characteristics of FP's to a single pulse of light of different lengths and also to a sequence of regular and coherent pulses of light.

I. FABRY-PEROT INTERFEROMETER

Figure 1a shows the series of beams produced by multiple reflections from the incident parent beam. When these beams interfere they give rise to the FP fringes, the amplitude of which is[1]

$$A = \sum_{n=0}^{\infty} TR^n e^{in\phi} = T/(1 - Re^{i\phi}) , \tag{1}$$

where T and R are flux transmittance and reflectance, respectively, and ϕ is the phase difference between the consecutive beams given by

$$\phi = (2\pi\nu/c)2d\cos\theta , \tag{2}$$

where d is the separation between the mirrors and θ is the angle of incidence of the parent beam (neglecting any change of phase on reflection). If the parent beam is collimated and is normally incident ($\theta = 0$) on the FP, the series of multiply reflected beams interferes. We describe this situation as the FP in filter mode because only a narrow band of frequencies can pass through the FP in this mode. If the incident beam is at an angle, the series of multiply reflected beams will interfere and form fringes at the focal plane of a lens that follows the FP. We describe this situation as the FP in fringe mode. The irradiance distribution in a fringe in either mode is given by the Airy curve, which is the square of the modulus of Eq. (1),

$$I = \mathcal{T}/[1 + F\sin^2\phi/2] , \tag{3}$$

where

$$\mathcal{T} \equiv [T/(1-R)]^2 \text{ and } F \equiv 4R/(1-R)^2 . \tag{4}$$

The maximum and minimum irradiances are (see Fig. 1b)

$$I_{max} = T/(1-R)^2 , \quad I_{min} = T/(1+R)^2 . \tag{5}$$

An important parameter of a FP is the finesse, N, which is directly proportional to its resolving power and is defined as the ratio of the free spectral range, $\Delta\phi$, i.e., the separation between the consecutive peaks, to the half-width of the Airy curve, $\delta\phi$ (see Fig. 1b),

$$N = \Delta\phi/\delta\phi = (\pi/2)\sqrt{F} = \pi\sqrt{R}/(1-R) , \tag{6}$$

where

$$\delta\phi = 2(1-R)/\sqrt{R} . \tag{7}$$

The free spectral range in terms of frequency is obtained from Eq. (2), since $\Delta\phi = 2\pi$,

$$\Delta\nu_{fsr} = c/2d\cos\theta . \tag{8}$$

Equation (1) refers to an infinite sum. But, we know that the amplitudes of the successive beams reduce ac-

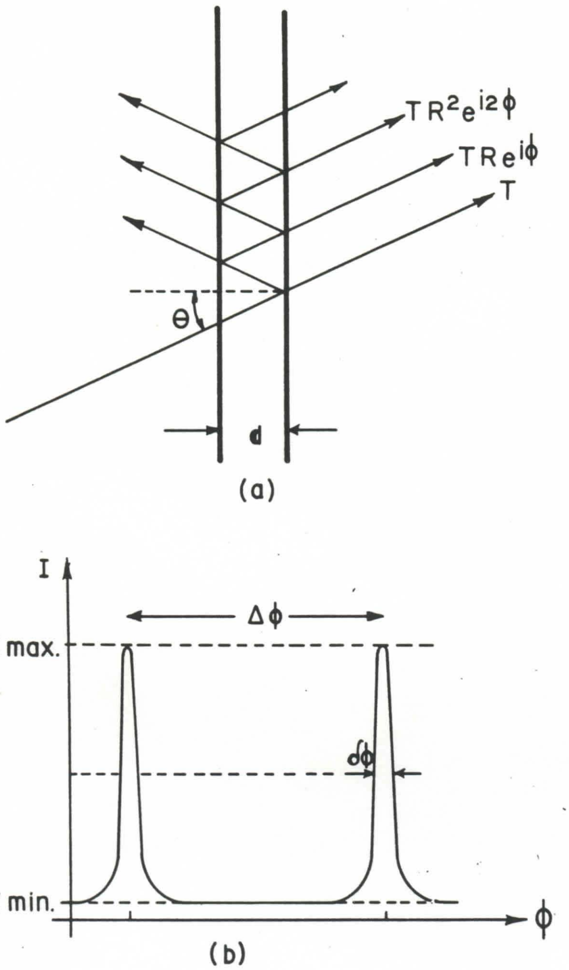

FIG. 1. (a) Multiple reflections in a Fabry–Perot interferometer (FP) formed by two plane parallel and partially transmitting mirrors of identical nature. (b) The transmission characteristics of a FP, known as the Airy curve.

cording to a geometric series. Thus, the beams, say after the Mth, will have negligible energy to alter appreciably the effect of the first M beams. Then Eq. (3) can be rewritten as

$$I=\mathcal{T}/[1+F\sin^2\phi/2]\approx I_M=\left|\sum_{n=0}^{M-1} TR^n e^{in\phi}\right|^2 . \tag{9}$$

We can find by computation that when the number M is of the order of N, the finesse, the approximation in Eq. (9) is quite good. As a matter of fact, when

$$M\approx 2N, \tag{10}$$

I_M is close to the ideal value for a wide range of values of R for mirrors flat to $\lambda/200$ or less.[2]

We shall now define two terms that will be used in the following sections. If the FP is in filter mode (the incident beam is collimated and normal to the FP), the transit time or delay time, τ_0, between the consecutive beams is

$$\tau_0=2d/c=1/\Delta\nu_{fsr} . \tag{11}$$

Because, as already mentioned, only the first M consecutive beams are required to produce the characteristic FP fringes and because M is of the order of N, the finesse, we define the response time of an FP as

$$\tau_r=N\tau_0 . \tag{12}$$

When the FP is in the fringe mode, the corresponding terms are:

$$\text{delay time,}\quad \tau_{0\theta}=2d\cos\theta/c=\tau_0\cos\theta , \tag{13a}$$

$$\text{response time,}\quad \tau_{r\theta}=N\tau_0\cos\theta . \tag{13b}$$

The resolving power of a FP can be defined by use of either the Rayleigh criterion or the half-width criterion. We shall adopt the Rayleigh criterion for a FP, because the resolving power of gratings is also defined by use of this criterion; then we shall have the convenience of direct comparison between the two instruments, although there is not an order of magnitude difference between the two criteria. The result is given in Born and Wolf,[4]

$$\mathcal{R}\equiv\lambda/\delta\lambda=n(0.97N) , \tag{14}$$

where the order of interference,

$$n=2d\cos\theta/\lambda \tag{15}$$

and $0.97N$ is called the effective number of interfering beams, say, M. Then Eq. (14) can be rewritten

$$\mathcal{R}\simeq nM=(M\cdot 2d\cos\theta)/\lambda . \tag{16}$$

Accordingly, the resolving power is the number of wavelengths in the path difference between the first and the last interfering wavefronts.

Equation (16) can also be rewritten, using Eq. (11), as

$$\mathcal{R}\equiv\nu/\delta\nu=\nu M\tau_0\cos\theta , \tag{17}$$

where $M\tau_0\cos\theta$ is the time difference between the first and the last interfering wavefronts produced by the incident pulse of length δt. Then we must have

$$\delta t\geqslant M\tau_0\cos\theta=1/\delta\nu$$

or

$$\delta\nu\delta t\geqslant 1 . \tag{18}$$

It is obvious that even if we have a potentially very-high-resolving-power FP, the effective resolution of an extremely short pulse, δt, will be limited to $\nu M\tau_0\cos\theta$, where M is

$$M\leqslant\delta t/\tau_0\cos\theta .$$

Or, conversely, the longer the pulse duration, the greater will be the resolution, provided that the instrument has the potential for it.

In the following sections, we shall analyze some of the temporal behaviors of an FP, beginning with the ideal situation in which the incident beam is an infinitely long wave train.

II. INFINITELY LONG WAVE TRAIN

A. Fabry-Perot interferometer in filter mode

We consider the situation in which the parent wave train is collimated and incident normally and has in-

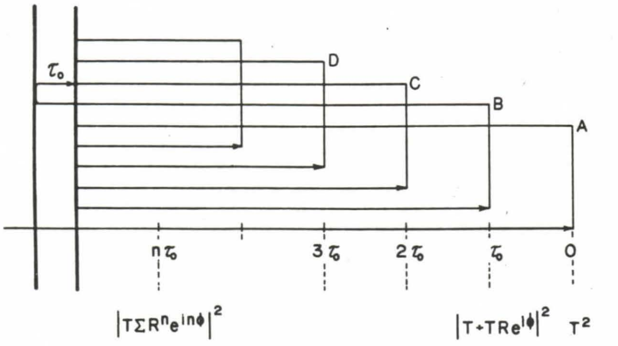

FIG. 2. Multitudes of wave trains (A, B, C, ...) of decreasing amplitudes but of the same delay time, τ_0, between each other are formed by a pair of parallel and partially transmitting mirrors. τ_0, called the delay time (see text), is the round trip time for light between the two mirrors. The resultant irradiance at a moment $n\tau_0$ is $T^2 \mid \Sigma R^n e^{in\phi} \mid^2$.

finitely long duration (as from a stabilized laser). Suppose the wavefront of this beam is designated by A (see Fig. 2). Then, at the output side of the FP, we shall have a series of wavefronts denoted by A, B, C, ... , the time delay between any pair of them being τ_0. With the origin of time at wavefront A defined as $t=0$, a detector will detect a varying irradiance that will reach the limiting value of I_{max} or I_{min} (see Fig. 3a) exponentially[3] but following a discrete staircase wave of periodicity τ_0. The over-all height of the mth step is produced by the first m consecutive beams,

$$I_m = \left| \sum_{n=0}^{m-1} TR^n e^{in\phi} \right|^2 = \mathcal{T}_m \cdot \frac{1+F_m \sin^2(m\phi/2)}{1+F\sin^2(\phi/2)}, \tag{19}$$

where

$$\mathcal{T}_m \equiv [T(1-R^m)/(1-R)]^2 < \mathcal{T} \tag{20a}$$

and

$$F_m \equiv 4R^m/(1-R^m)^2 < F . \tag{20b}$$

Equation (19) is similar to the one that describes the fringes due to a Lummer–Gehrke interferometer.[4] The half-width of these fringes, $\delta\phi_m$, is greater than that for the ideal Airy curve, $\delta\phi$, especially, when $m<N$ and R is large. The exact expression is[4]

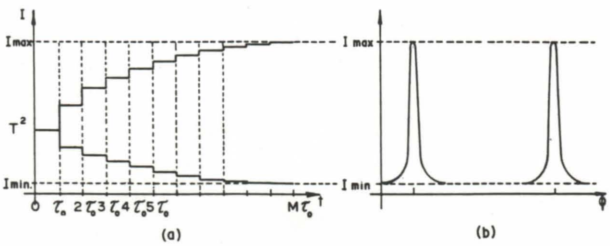

FIG. 3. (a) Temporal development of the transmission characteristics of a Fabry–Perot (FP): the up-going staircase indicates the development when the FP is in transmission mode and the down-going staircase indicates the development when the FP is in reflection mode. τ_0 is the FP delay time. (b) For the sake of comparison, the steady-state transmission characteristics of an FP (the Airy curve) is shown alongside.

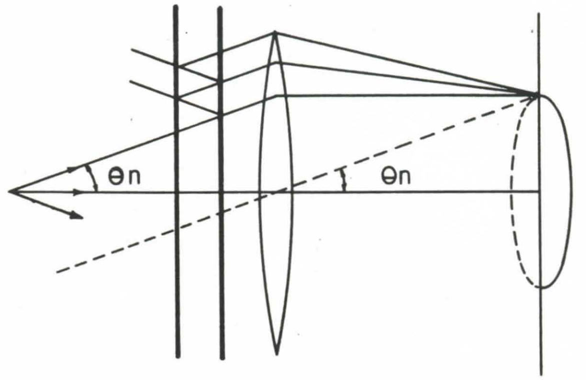

FIG. 4. A Fabry–Perot interferometer in fringe mode (formation of the fringes with an incident divergent beam).

$$\sin^2(m\,\delta\phi_m/4) - (F/2F_m)\sin^2(\delta\phi_m/4) + (1/2\,F_m) = 0. \tag{21}$$

Then the finesse is given by

$$N_m = \Delta\phi_m/\delta\phi_m \tag{22}$$

or, for high reflectivity[4]

$$N_m \simeq m . \tag{23}$$

The irradiance difference between any two consecutive steps (Fig. 3a) will be

$$\left| I_n - I_{n+1} \right| = T^2 \left| \left| \sum_{q=0}^{n-1} R^q e^{iq\phi} \right|^2 - \left| \sum_{q=0}^{n} R^q e^{iq\phi} \right|^2 \right| . \tag{24}$$

The usefulness of such a fast, exponentially rising, staircase wave is not apparent (for $d=1$ cm, $\tau_0=66$ ps). Nevertheless, Fig. 3a now makes it clear why it is necessary to wait for at least a time $\tau_r = M\tau_0$ after the arrival of the first transmitted wavefront to register an interference effect (Airy curve) characteristic of an FP; Fig. 3b shows a normal Airy curve for convenience of comparison.

B. Fabry-Perot interferometer in fringe mode

A FP in the fringe mode is sketched in Fig. 4. To simplify the analysis, let us assume that we have a point source that illuminates the FP from a distance such that it is at the focal point of the lens behind the FP that focuses the fringes in its focal plane. This condition implies that the first transmitted wavefront from the divergent beam will be collimated by the lens and hence arrive at the entire fringe plane simultaneously. The set of rays that forms the nth fringe at an angle θ will have a time delay of $\tau_0 \cos\theta$ Eq. (13a) between any consecutive pair of the set. Therefore, the response time, $\tau_{r\theta}$, for the formation of the nth fringe is $M\tau_0\cos\theta$, in contrast to $M\tau_0$ for the central fringe ($\theta=0$). Furthermore, because

$$M\tau_0 \cos\theta < M\tau_0 \tag{25}$$

the outermost fringe is completely formed first and the central fringe last. Similarly, if the incident beam is of finite duration, the central spot will tend to disappear last. Therefore, to a very fast detector, the fringes may appear to walk. When the point source is at a distance other than the focal length of the lens, the first transmitted wavefront will be spherical, hence the different parts will arrive at the fringe-forming plane at different times. This relative time delay is to be taken into account to find the direction in which the fringes walk.

III. A LONG PULSE OF FINITE LENGTH

In this section, we shall find the response of an FP to a long rectangular pulse of light. Let us assume that the temporal duration of the individual pulses, δt, is long compared to the delay time, τ_0, of the FP, so that multiple beams are produced for interference;

$$\delta t > \tau_0 \ . \tag{26}$$

On the basis of the order of magnitude relation,[5] formed by the product of the half widths of two functions (one of ν and the other of t) that form a Fourier-transform pair,

$$\delta\nu\delta t \sim 1 \tag{27}$$

and of Eq. (11), Eq. (26) can be rewritten in terms of frequency,

$$\Delta\nu_{fsr} > \delta\nu \ , \tag{28}$$

where $\delta\nu$ is the spectral width of the incident radiation. Thus, to have a regular interferometric response, we must have the dynamic condition, Eq. (26), which is equivalent to the well-known static condition, Eq. (28), that the free spectral range should be larger than the width of the spectrum to be analyzed.

To return to the response of an FP to such long pulses, the initial development can be understood from Fig. 3. The transmitted irradiance follows the staircase wave, Eq. (19), but if

$$\delta t = m\tau_0 \tag{29}$$

and

$$m\tau_0 < M\tau_0 \ , \tag{30}$$

where M is the effective limiting value of the number of beams defined in Eq. (9), the asymptotic value corresponding to the Airy curve can never be reached. The situation is depicted in Fig. 5, where the pulse length is four times the FP delay time; consequently no more than four consecutive reflected beams can interfere at a time. The extreme value of irradiance will be reached at a moment $t = m\tau_0$ s after the reception of the first transmitted wavefront (A in Fig. 5). From this moment, onward, the shape of the irradiance curve is given by Eq. (19). However, the peak irradiance will decrease steadily because the order of reflection of the series of interfering beams will increase, reducing their amplitudes in geometric progression, while the number of beams in the series m will remain the same, implying that the finesse will remain constant,[6] N_m. This can be appreciated from Eq. (19), where m is fixed but n starts from values higher than zero.

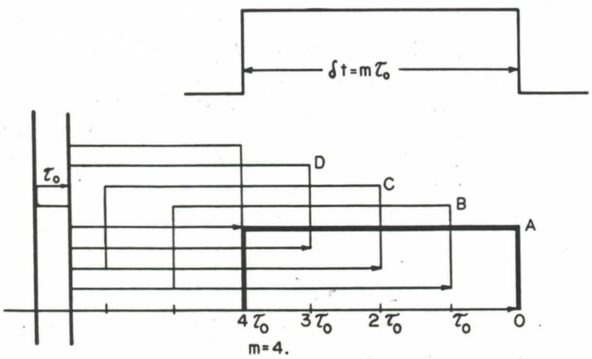

FIG. 5. Of the multitudes of pulses formed by a Fabry–Perot, only a limited number of them can interfere when the incident pulse is of finite length. The upper rectangular curve indicates the length of the incident pulse. τ_0 is the Fabry–Perot delay time.

If the pulse length is such that

$$\delta t = m\tau_0 > M\tau_0 \ , \tag{31}$$

the resultant irradiance curve is given effectively by the ideal Airy curve, Eq. (9). Consequently, a FP cannot distinguish spectroscopically between pulses of duration $M\tau_0$ and infinity. If we are certain that the duration of the wave train is infinite (perfectly monochromatic radiation), we take the Airy curve as the instrumental response curve rather than the actual spectral distribution (which is a delta function). When we have a finite pulse, to avoid the ambiguity, we should set the FP separation such that the delay time, τ_0, gives rise to an effective number of interfering beams, m, less than M, as in Eq. (30). Although m can be controlled by τ_0, the FP has an uncertainty about the exact value of m because we are trying to analyze a spectrum of unknown duration δt [see Eq. (29)]. However, δt (and hence $\delta\nu$) can be estimated by means of several experiments, by setting the FP for different known values of τ_0. A grating would be preferable because the number of beams, m (number of lines in the grating), can be controlled precisely.

To produce interference effects, according to Eq. (26) the pulse length δt of the light should be greater than the delay time, τ_0. The pulse length should also be less than $M\tau_0$ to avoid the range of ineffectiveness (ambiguity) of the FP. When the two conditions are put together, we get, in the time and frequency domains,

$$M\tau_0 > \delta t > \tau_0 \tag{32a}$$

and

$$\Delta\nu_{fsr}/M < \delta\nu < \Delta\nu_{fsr} \ . \tag{32b}$$

IV. A SHORT PULSE

We shall define a short pulse as one whose temporal duration is shorter than the delay time of the FP, i.e., when

$$\delta t < \tau_0 \tag{33}$$

Then the FP produces a chain of transmitted pulses of decreasing amplitude with separation τ_0 between them. However, they do not overlap each other and hence produce absolutely no interference effect (see Fig. 6). Therefore, no spectroscopy of a pulse shorter than the FP delay time can be performed with an FP. For example, spectroscopy with 0.2 ps pulse would require a

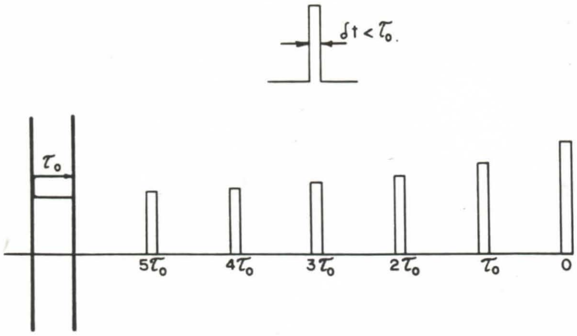

FIG. 6. A pulse of length shorter than the Fabry–Perot delay time, τ_0, cannot produce normal interference effects, since the generated pulses are separated from each other. The upper rectangular curve indicates the length of the incident pulse.

FP of separation $d(= c\tau_0/2)$, much smaller than 30 μm, which is difficult to fabricate. Besides, the resolving power of the FP would be very small because it is directly proportional to the product of the finesse and the order of interference $(2d/\lambda)$. A grating would also suffer from the same low resolution owing to the short length of the pulse. However, producing multiple beams for interference is much easier for gratings in this domain of short pulses because the delay time of a grating at the nth diffraction order is $2n\times10^{-15}$ s at $\lambda = 6000$ Å (for a grating $\tau_0 = n/\nu$ and $\tau_r = Nn/\nu$, where N is the total number of slits and n is the order of diffraction).

In any case, for an infinitely short pulse, the conventional definition indicates zero resolution, because no multiple-beam interference can be produced with a single FP or grating. This accords very well with the product relation of the half-widths of two functions which are a Fourier-transform pair, as in Eq. (27),

$$\delta\nu\delta t \sim 1 \; . \tag{34}$$

In Sec. VI we shall attempt to show the difficulty of doing spectroscopy with extremely short pulses even using two FP's.

V. SERIES OF COHERENT SHORT PULSES

A. Fabry–Perot delay time equals pulse separation

A regular series of coherent pulses can be obtained from mode-locked lasers. The exact shape of such pulses is determined from the number of longitudinal modes and their relative strengths. Here we shall consider an idealized situation of rectangular pulses, as in previous sections. First, we shall consider the case where the FP delay time equals the regular pulse separation,

$$\Delta t = \tau_0 \; . \tag{35}$$

The pulse width, δt, is assumed to be very small. The situation is depicted on the top line of Fig. 7. By multiple reflection, each of these pulses produces a chain of nonoverlapping pulses of decreasing amplitude as shown in Fig. 7. Because all of these pulses are coherent, the overlapping pulses that belong to different chains interfere.

Figure 7 permits a detailed examination. The chain of pulses on lines 0, 1, 2, ... are produced by the 0th, 1st, 2nd, ... pulses, respectively, of the original incident series of pulses. As a particular case, we have indicated the moment $3\tau_0$ with a heavy arrow at the bottom of Fig. 7. All of the four pulses that fall along the indicated dotted vertical line interfere. In general, at moment $m\tau_0$, the resultant irradiance is

$$I_{m+1} = \left| \sum_{n=0}^{m} TR^n e^{in\phi} \right|^2 . \tag{36}$$

Or, when $m \geqslant M$, the Eq. (36) leads to the ideal Airy function of Eq. (9). So, a slowly scanning FP should reproduce the spectrum of all of the longitudinal modes of the original mode-locked laser pulses. Of course, each longitudinal mode would be widened by the instrumental Airy function.

It is clear that exact matching of the FP delay time with the imprecisely known pulse separation, Δt, would be an extremely difficult operation. But if the FP is set for a τ_0 that is slightly larger than Δt, then it can be used in the fringe mode to find the spectrum. Suppose that we get a perfect matching of

$$\Delta t = \tau_0 \cos\theta_n \tag{37}$$

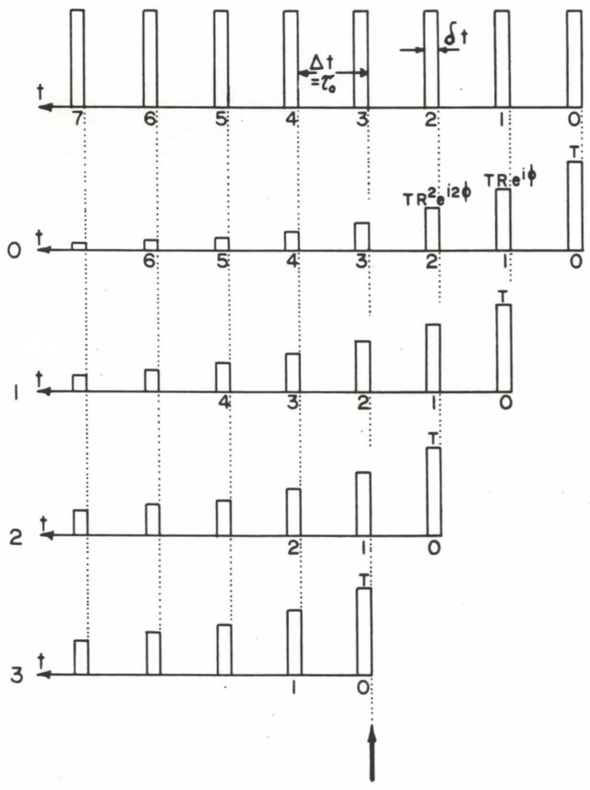

FIG. 7. A series of coherent short pulses of separation equal to the Fabry–Perot delay time, τ_0, can produce regular interference effects. The top row indicates the incident series of pulses. The following rows indicate the train of pulses produced by the 0th, 1st, 2nd, ... incident pulses on matching time scales. The overlapping pulses at a particular moment are indicated by a heavy arrow at the bottom. The amplitudes and the phases of the pulses are indicated above each one.

at an angle θ_n, where

$$2d\cos\theta_n = n\lambda = nc/\nu \tag{38a}$$

or

$$\tau_0 \cos\theta_n = n/\nu \tag{38b}$$

or, combining the above equations,

$$\Delta t = \tau_0 \cos\theta_n = n/\nu \ . \tag{39}$$

Then, only the nth order spectrum at an angle θ_n will be formed as a perfect FP fringe. At an angle $\theta_{n\pm p}$ where the $(n\pm p)$th-order fringe should have formed under a steady-state condition, we have

$$\tau_0 \cos\theta_{n\pm p} = (n\pm p)/\nu = (n/\nu) \pm (p/\nu) \ . \tag{40}$$

Thus, at angle $\theta_{n\pm p}$, the centers of the successive interfering pulses will be advanced or retarded by $(\pm p/\nu)$ seconds. Then, when the width of the rectangular pulse is δt, the $(n\pm p)$th-order fringe will be formed by interference of only m wavefronts given by,

$$m = \delta t/(p/\nu) \ . \tag{41}$$

Consider a typical example

$$\delta t = 5\times 10^{-13}\ \text{s} \quad \text{at } \nu = 6\times 10^{14}\ \text{c/s and } p\sim 10 \ .$$

Then

$$m = \delta t\nu/p = 30 \ .$$

For a high-finesse FP ($N\sim 70$), the fringes produced by only 30 wavefronts will be clearly detectable ($N_{30}\sim 30$). Thus, from Eq. (40) along with Eq. (19) we can estimate the width of very sharp pulses. In fact, the narrower the pulse, the better will be the estimate.

In reality, pulses are never exactly rectangular. So, the actual shape of the pulses will have to be taken into account in rigorous computation.

B. Pulse separation is an integral multiple of the delay time

We now consider a situation in which the coherent regular pulses have a separation that is an integral multiple of the FP delay time,

$$\Delta t = q\tau_0 \ . \tag{42}$$

But the pulse width, δt, is still smaller than τ_0. The situation is depicted in Fig. 8, where a particular case of $\Delta t = 4\tau_0$ is shown. Now consider the moment $20\tau_0$ after the first transmitted pulse has been received (the heavy arrow at the bottom). Then, the interfering wavefronts constitute the pulses: (5-0), (4-4), (3-8), (2-12), (1-16), and (0-20). The resultant intensity is given by

$$I = \left| T + TR^4 e^{i4\phi} + \cdots + TR^{20} e^{i20\phi} \right|^2 ,$$

where the number of overlapping wavefronts is $(20/4+1)$. Or, in general, at a moment $m\tau_0$, the resultant intensity is given by

$$I_{(m/q+1)} = \left| \sum_{n=0}^{m/q} TR^{nq} e^{inq\phi} \right|^2 , \tag{43}$$

where the number of overlapping wavefronts is $(m/q+1)$. After a long time, or when $m \geq M$, the series can be considered to be effectively infinite and then using the similarity with Eqs. (1), (3) and (4), we can write

$$I = \mathcal{T}_q/[1 + F_q \sin^2 q\phi/2] \ , \tag{44}$$

where

$$\mathcal{T}_q \equiv T^2/(1-R^q)^2 < \mathcal{T} \ , \tag{45}$$

$$F_q \equiv 4R/(1-R^q)^2 < F \ , \tag{46}$$

and the finesse

$$N_q = \frac{1}{\sqrt{q}} \cdot \frac{\pi}{2}\sqrt{F_q} \ . \tag{47}$$

Equation (44) clearly shows a decrease of the free spectral range, which is now

$$\Delta\nu_{fsr,q} = \Delta\nu_{fsr}/q = 1/q\tau_0 \ . \tag{48}$$

But, because the spectral width from Eq. (42) is

$$\Delta\nu \sim 1/\Delta t = 1/q\tau_0 \ , \tag{49}$$

there will be no overlap of different-order spectra and once again the spectroscopy of such coherent pulses can be carried out with a suitable FP. But the widening of the spectra is not due to the regular Airy function, given by Eq. (3); rather it is due to the new function given by Eq. (44).

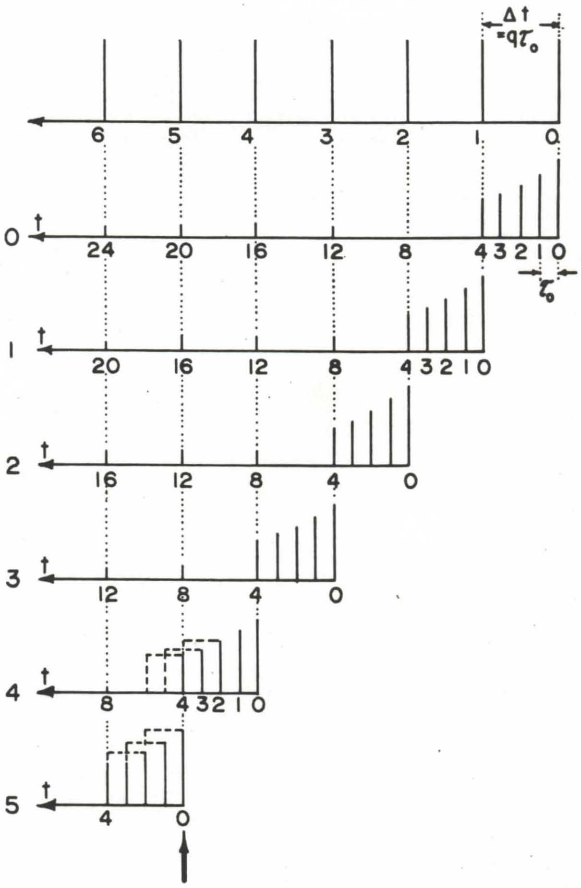

FIG. 8. A situation that is very similar to that of Fig. 7 but with pulse separation equal to an integral multiple of the Fabry–Perot delay time, τ_0. The broken lines on the bottom two rows indicate the overlapping of the adjacent pulses when the width is larger than τ_0.

Estimation of the pulse width with an FP under these circumstances, by use of it in the fringe mode, can be attempted as we have analyzed at the end of Sec. V A, but the reduction of the new finesse, N_q, will reduce the accuracy of estimation compared to the previous situation.

A slightly different situation can also be considered. Let the FP be in the filter mode, to avoid complications. Let the pulse separation be, as before, q times the delay time, Eq. (42), but let the pulse width be more than the delay time. First, we shall consider a particular case, $\delta t = 2\tau_0$, but $\Delta t = 4\tau_0$, as before. The situation can be realized by looking at the bottom part of Fig. 8, where the broken lines show the width of the pulses. Then the interfering wavefronts at moment $5\tau_0$ are, (5-0), (4-4, 3, 2), (3-8, 7, 6), (2-12, 11, 10), (1-16, 15, 14), (0-20, 19, 18) instead of just (5-0), (4-4), (3-8), (2-12), (1-16), and (0-20), as before.

So, we can find the resultant irradiance at the moment $5\tau_0$,

$$I = \Big| T + TR^4 e^{i4\phi}(1 + R^4 e^{i4\phi} + \cdots + R^{16} e^{i16\phi}) + TR^3 e^{i3\phi}(1 + R^4 e^{i4\phi} + \cdots + R^{16} e^{i16\phi}) + TR^2 e^{i2\phi}(1 + R^4 e^{i4\phi} + \cdots + R^{16} e^{i16\phi}) \Big|^2 .$$

Or, in general, when the pulse separation and width are given by

$$\Delta t = q\tau_0 \quad \text{and} \quad t = s\tau_0 \tag{50}$$

(note that $s < q$ is necessary for original non-overlapping pulses), we would find the resultant irradiance at a moment $m\tau_0$,

$$I = \Big| T + T(R^q e^{iq\phi} + R^{(q-1)} e^{i(q-1)\phi} + \cdots + R^{(q-s)} e^{i(q-s)\phi}) \times (1 + R^q e^{iq\phi} + \cdots + R^{q(m-1)} e^{iq(m-1)\phi}) \Big|^2$$

or,

$$I = \left| T + TR^{q-s} e^{i(q-s)\phi} \cdot \frac{1 - R^{(s+1)} e^{i(s+1)\phi}}{1 - R e^{i\phi}} \cdot \frac{1 - R^{qm} e^{iqm\phi}}{1 - R^q e^{iq\phi}} \right|^2 . \tag{51}$$

This is the general expression for the instrumental function, which needs detailed computation to find the finesse and the nature of the curve. But, we shall consider a special situation in which the width of the pulse is such that

$$(q - s) = 1 . \tag{52}$$

Careful scrutiny will show that with this condition, the series of interfering wavefronts constitutes the regular FP series. Either this physical argument or straightforward algebra reduces Eq. (51) to,

$$I = \left| T \cdot \frac{1 - R^{(qm+1)} e^{i(qm+1)\phi}}{1 - R e^{i}} \right|^2 . \tag{53}$$

With the additional feasible assumption that $(qm + 1) \geq M$ implying $R^{(qm+1)} \simeq 0$, we have [see Eq. (3)]

$$I = \mathcal{T}/[1 + F \sin^2\phi/2] . \tag{54}$$

Surprisingly, this is again the ideal Airy function, a situation similar to that which we have come across before in Sec. V A. Thus, with the proper matching conditions given by Eqs. (50) and (52), we can again do spectroscopy of coherent pulses where the instrumental function is exactly the ideal Airy function. Such conditions can also be used with the FP in fringe mode, as has been described at the end of Sec. V A.

VI. A SHORT PULSE THROUGH TWO FP'S

In Sec. IV, we considered the case of one short pulse or incoherent short pulses through one FP and we found that spectral analysis could not be done because the multiply reflected pulses do not overlap and interfere to produce the spectrum of the incident light. In this section, we shall attempt to see whether any interference at all can be generated by using two FP's instead of one. The top part of Fig. 9 shows a chain of nonoverlapping pulses generated by the first FP from the incident single pulse. Each of these pulses will generate a chain of pulses through the second FP as shown on the lower lines of Fig. 9. If the delay times are appropriately matched, we can again see interference of many wavefronts and hence spectroscopy should be possible.

First, consider the situation when both the FP's have exactly the same delay time. This can also be achieved by using one FP, by sending the pulses back through the

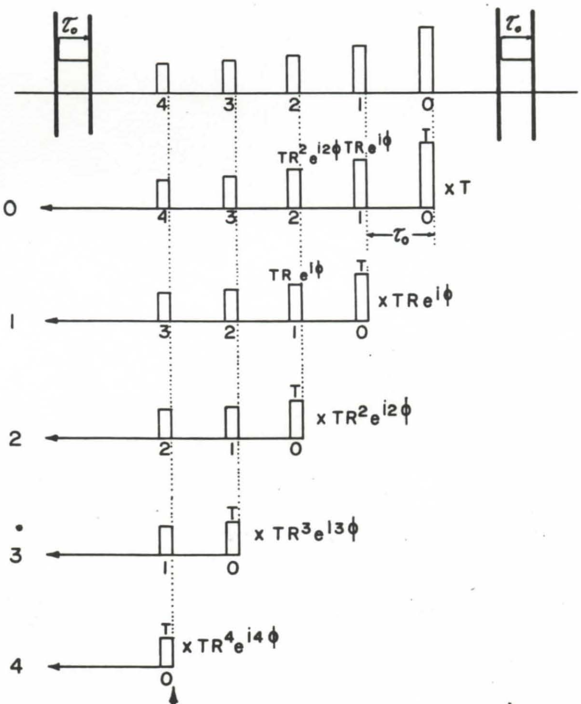

FIG. 9. The possibility of interference with a single but very short pulse using two identical Fabry–Perots. Top row: A single short pulse produces a series of non-overlapping pulses through the first Fabry–Perot (FP). These pulses, in turn, pass through the second FP and produce many trains of pulses, which are indicated in the following rows on matched time scales. The heavy arrow at the bottom indicates the overlapping pulses at a particular moment. The amplitude and phase of any pulse are obtained by multiplying the expression shown above it by the factor that is indicated on the extreme right of the corresponding row.

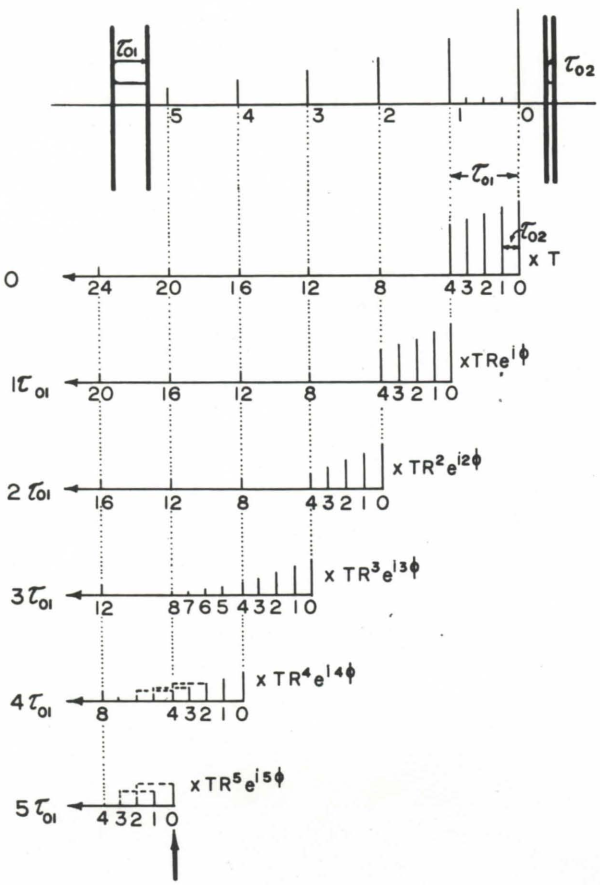

FIG. 10. A situation that is very similar to that of Fig. 9, but with two Fabry–Perots whose delay times are integral multiples of each other ($\tau_{01} = 4\tau_{02}$). The broken lines at the bottom two rows indicate the overlapping of the adjacent pulses when the width is larger than τ_{02} but narrower than τ_{01}.

same FP, by use of a corner cube or other suitable optics (this device is often called a double-passed FP). If we wait until the mth pulse, produced by the first FP, passes through the second FP, then the resultant intensity should be

$$I = |T^2R^m e^{im\phi} + T^2R^m e^{im\phi} + \cdots,\ (m+1)\text{terms}|^2$$
$$= (m+1)^2T^2|TR^m e^{im\phi}|^2 . \tag{55}$$

All of the interfering pulses, being of exactly the same phase, no dispersion effect is produced and so no spectroscopy is possible.

Let us now consider a situation in which the second FP has a smaller delay time than the first one,

$$\tau_{01} = q\tau_{02} . \tag{56}$$

This amounts to a situation in which we have a chain of coherent pulses produced by the first FP of separation

$$\Delta t \equiv \tau_{01} = q\tau_{02} . \tag{57}$$

Then the situation is similar to that depicted in Fig. 8, with the difference that the successive pulses that enter the second FP have amplitudes decreasing in geometric progression. It has been redrawn in Fig. 10. We are still considering very narrow pulses, such that the width $\delta t < \tau_{02}$. But, now we have two FP's with different separations and so the phase delays ϕ for the first and ϕ' for the second are also different. Using Eqs. (2) and (11) and considering normal incidence, we have

$$\phi = 2\pi\nu\tau_{01} \quad \text{and} \quad \phi' = 2\pi\nu\tau_{02}. \tag{58}$$

Then considering a particular case of

$$\tau_{01} = 4\tau_{02}, \tag{59}$$

the interfering irradiance at a moment $5\tau_{01}$ (indicated by the heavy arrow in Fig. 10) is

$$I = |T^2R^5e^{i5\phi}[1 + R^3e^{i\Phi} + R^6e^{i2\Phi} + \cdots + R^{i15}e^{i15\Phi}]|^2 ,$$

where $\Phi \equiv 4\phi' - \phi = 0$ [using Eqs. (58) and (59)]. Or, in general, at a moment $m\tau_{01}$ with $\tau_{01} = q\tau_{02}$,

$$I = \left| T^2R^m e^{im\phi} \sum_{n=0}^{m} R^{(q-1)n} e^{in\Phi} \right|^2 , \tag{60}$$

where

$$\Phi \equiv q\phi' - \phi,$$

or, using Eqs. (58) and (57),

$$\Phi = 2\pi\nu(q\tau_{02} - \tau_{01}) = 0 . \tag{61}$$

Then, due to Eq. (61), Eq. (60) becomes

$$I = T^4R^{2m} \left| \frac{1 - R^{(q-1)(m+1)}}{1 - R^{(q-1)}} \right|^2 . \tag{62}$$

Again as in Eq. (55), we have lost the dispersion effect and hence the possibility of spectral analysis. Thus, we are forced to conclude that the superposition of identical parts through (passive) replication from a single short pulse does not produce any information regarding its frequency composition. In the strictest sense, interference is taking place. But does it have any physical meaning from the view point of an experimentalist? It is apparent that the superposition of physically different parts of a wave train is a necessary condition for producing a dispersion effect (frequency information) through interference (diffraction).

We have also considered a situation that is somewhat similar to the case considered at the end of Sec. V.B, where the width of the incident pulse is larger than that of the second FP; $\tau_{01} = q\tau_{02}$ and $\delta t = s\tau_{02}$. The situation can be understood from Fig. 10 (note, especially, the dotted curves which indicate the width of the pulses). Again the analysis confirms the above conclusions. The essential dispersive effect from interference arises due to the superposition of the different physical parts of the same wave train. Thus, the proper spectral analysis of a single ultrashort pulse is extremely difficult, if not impossible, by interferometry.

VII. DISCUSSION

We have derived the response characteristics of Fabry–Perot interferometers for a single pulse of light of different lengths and also to a sequence of regular and coherent pulses of light. It appears that the width of such pulses can be determined and their spectral analysis can be carried out with a judicious choice of the separation between the Fabry–Perot plates for pulses of width down to subnanoseconds, whereas such analysis is extremely difficult, if not impossible, for pulses of width of the picosecond range by interferometry.

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