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Bi-centenary of successes of Fourier theorem! Its power and limitations in optical system designs.

Chandrasekhar Roychoudhuri
Photonics, Lab., Physics Department, U. Connecticut and Femto Macro Continuum Storrs.

Abstract

We celebrate the two hundred years of successful use of the Fourier theorem in optics. However, there is a great enigma associated with the Fourier transform integral. It is one of the most pervasively productive and useful tool of physics and optics because its foundation is based on the superposition of harmonic functions and yet we have never declared it as a principle of physics for valid reasons. And, yet there are a good number of situations where we pretend it to be equivalent to the superposition principle of physics, creating epistemological problems of enormous magnitude. The purpose of the paper is to elucidate the problems while underscoring the successes and the elegance of the Fourier theorem, which are not explicitly discussed in the literature. We will make our point by taking six major engineering fields of optics and show in each case why it works and under what restricted conditions by bringing in the relevant physics principles. The fields are (i) optical signal processing, (ii) Fourier transform spectrometry, (iii) classical spectrometry of pulsed light, (iv) coherence theory, (v) laser mode locking and (vi) pulse broadening. We underscore that mathematical Fourier frequencies, not being physical frequencies, cannot generate real physical effects on our detectors. Appreciation of this fundamental issue will open up ways to be innovative in many new optical instrument designs. We underscore the importance of always validating our design platforms based on valid physics principles (actual processes undergoing in nature) captured by an appropriate hypothesis based on diverse observations. This paper is a comprehensive view of the power and limitations of Fourier Transform by summarizing a series of SPIE conference papers presented during 2003-2007.

1. Introduction

The phrase "optical design" generally refers to simple and complex imaging systems utilizing mostly the Snell's law and some form of the Huygens-Fresnel Principle (HFP) to model the diffractive propagation of light through the optical system. Designing a simple high resolution imaging system requires understanding of the impulse response function based on spatial Fourier transforms (FT). Designing illumination systems for very high magnification microscopes require understanding of coherence theory which requires the use of both spatial and temporal Fourier transforms, the degree of temporal coherence function being the Fourier transform of the spectral density function, which itself could be the Fourier transform of the time-finite amplitude of the light signal. Designing a Fourier transform spectrometer requires an appreciation of both spatial and temporal coherence, which are again expressed as FT of different properties of light beams. Designing mode locked lasers and auto-correlators to characterize the short pulses and to propagate them through fibers or complex optical devices require appreciation of time-frequency FT relations. Photons in quantum electrodynamics are defined as the Fourier modes of the "vacuum" or the cosmic medium, as yet beyond our direct perception or analysis.

Thus, FT plays a profoundly deep role, almost like a principle of nature, in both classical and quantum optics, yet it is only a tool and not an accepted principle of nature. The purpose of this paper is to take a number of specific uses of FT in classical optics and illustrate how each one of the varied and specific forms of FT-use actually corresponds to different functional (accepted) principle of physics. It is worth the effort of optical system designers to recognize how the FT conjugate pair of variables corresponds to actual physical parameters in real instruments to appreciate the behavior of the nature of light and the limitations of the Fourier theorem. Appreciating these subtle limitations will open up new ways of designing optical systems in all the cases mentioned in the last paragraph.

Whether it is the Schrödinger's cat, Dirac's photon or a classical optical pulse or a spatial aperture, the mathematical technique of Fourier transform is simultaneously the most elegantly and most erroneously used theorem in the history of engineering and physical sciences! Because, many a times we forget to distinguish between physical and mathematical spaces ignoring the physical interactions required in the real world for real transformations that we measure by our instruments. It is important to ask a question like: When do the human logics behind various mathematical transforms mirror any of nature's actual processes, meaning, any of the cosmic logics in operation that we are really after? We have slowly abandoned seeking realities behind natural interaction processes in classical physics starting over a couple of centuries ago enamored by the successes provided by our mathematics. Quantum philosophies espousing "teleportation", "multiple universes", etc., are just an outgrowth of these classical practices!

2007 is the bicentennial celebration year for the first successful introduction of a theorem by Joseph (Jean Baptiste) Fourier (1768-1830) to solve the evolution of the effects due to various impulses on physical systems by expressing the physical impulse in terms of summation of finite or infinite number of harmonic functions. The power of this mathematical tool, the Fourier theorem, evolved slowly in multiple steps through a number of its successful uses. They are: (i) Derivation of the far-field diffraction pastern due to an aperture and consequent "optical signal processing" after recognition that an optical lens can also carry out "spatial" Fourier transform under specific conditions. In fact, Fourier transform like pattern always arises at the plane of convergence whose scaling factor and the quadratic phase factor may vary from case to case. This recognition has opened up the field of Optical Signal Processing that started in 1940's and accelerated from 1960's to a matured technology now. Mathematically this is a space-spce Fourier transform that maps the complex amplitude pattern of the optical field from one spatial plane to another spatial plane. The correctness of this mathematical approach is based on the fact that the natural propagation of light field has been embodied elegantly by Huygens-Fresnel principle and the corresponding integral morphs into a Fourier transform-like integral in the far-field or the plane of convergence in an optical instrument. (ii) Michelson recognized that different optical frequencies do not interfere with each on the beam splitter of his now-famous interferometer. So the multi frequency fringes can be expressed as summation of the fringe intensities due to each different frequency. So Michelson developed the algorithm of re-expressing quadratic $\cos^2$ fringes as a "DC" component plus a linear cosine. Thus, the "DC" free fringe intensity for multiple frequencies is simply a linear sum of the cosines of the individual frequencies. Fourier inversion of this "DC" free intensity gives the optical spectral distribution. During late 1800, this was the birth of an enormously important high resolution spectroscopic technique, named as Fourier Transform Spectroscopy (FTS), which is still one of the most important spectroscopic techniques in practice. FTS that relies on taking the Fourier transform of the visibility of Michelson interferometer fringes [*delay-frequency transform*]. (iii) classical spectrometry of pulsed light that expresses the spectral fringe as the convolution of the CW-impulse response with the Fourier spectral intensity due to the incident time pulse [*time-frequency transform*], (iv) laser pulse shape and spectral analysis based on coherence theory and the Wiener-Khintchine (Autocorrelation) theorem [delay-*frequency and time-frequency mixed transforms*], and (v) pulse broadening due to propagation through a material medium by propagating the Fourier component frequencies of the pulse through the material *[time-frequency transform and acceptance of Fourier frequencies as real physical*].

Our key point is that the key equations that supports design platforms for optical systems must be based on valid physics principles (actual processes undergoing in nature) captured by an appropriate hypothesis based on diverse observations. Otherwise we may accept erroneous concepts as valid principles of nature.

This paper is a summary of a series of papers presented mostly in SPIE conferences by the authors during 2003-2007 [1-22]. However, the author tried unsuccessfully during the 1970's [23-28] to draw attention of the community that the unquestioning application of the mathematical Fourier theorem, as if it is a physics principle, should be critically reviewed for the benefit of progress in optical science and technology. These publications are mostly in a local magazine.

2. What can an optical system designer take out of this Fourier transform related paper?

Let us underscore first why an optical system engineer might be interested spending the time to understand the mathematical subtleties behind Fourier transforms. First, in general, a deeper understanding of the applicable theory helps an engineer to become more innovative in improving and/or designing newer instruments. Second, various types of Fourier transforms with same mathematical structure but different types of transform-space-pairs cover a very wide range of optical instruments covering the entire matured field of optical technologies. Third, this paper will demonstrate that inspite of correctness of the FT as a mathematical theorem, we have been using it incorrectly in a few a places that we will underscore. This recognition opens up the possibilities for many new inventions and we will give some specific examples.

(i) Space-space transform; optical signal processing: FT-formalism driven optical signal processing is in a sound platform because the physics principle of Huygens-Fresnel Integral morphs into FT under right condition. Although one should be aware of pitfalls of imaging with ultra short light pulses, it is not a serious problem for imaging applications since the relative delays in the image plane is essentially zero [3, 5, 27, 29].

(ii) Delay-frequency transform; Fourier transform spectroscopy (FTS): FTS is also on a sound platform as long as one does not use (i) fast detector and (ii) the maximum interferometer delay is smaller than the pulse width. Otherwise, differential amplitude induced visibility reduction would artificially broaden the recovered spectrum [16, 25].

(iii) Time-frequency transform, classical spectrometry: Classical spectrometry also gives numerically correct results but only for light pulses that are definitely longer than the instrument's characteristic time constant, $\tau_0 = R\lambda / c$, R being the classical resolving power. For some unknown reasons, this time constant is not explicitly recognized in classical spectrometry. We will show that the true spectrometer impulse response must be derived by time domain propagation of a pulse that can converges to classical CW formulation for signal duration longer than this time constant. Time integrated fringe broadening due to a pulse do correspond to convolution of the CW impulse response with the Fourier intensity spectrum by virtue of conservation of energy (Parseval's theorem). Recognition of this subtlety tells us that the traditionally accepted time-frequency bandwidth limit $\delta t \delta \nu \geq 1$ is not a fundamental principle of nature, which opens up the door to designing algorithm and instruments to achieve spectral super resolution. We will give summary of the necessary derivation and some experimental results [8, 14, 21, 23-24, 26].

(iv) Time-frequency transform, Coherence theory: First let us appreciate that all light signals must necessarily be time and space finite pulse dictated by the principle of conservation of energy. Even a CW laser has to be turned on and off in the real world. The physical spectrum of a pulse is its actual carrier frequencies (undulations of the E &B field vectors) contained in it, not the FT of the amplitude envelope. Fringe visibility (autocorrelation function) can be degraded by unequal amplitudes of same frequency light pulse and also by displaced fringe intensities due to different frequency light of long duration. Ignoring this equivalency of the experimental visibility has led us not to distinguish between temporal coherence (due to a time finite pulse with a single carrier frequency) and spectral coherence (due to CW or pulsed light that inherently contains multiple carrier frequencies). We will give simple mathematical derivations to underscore our points. These understandings will provide the platform for proper characterization of ultra short light pulses whose spectral content (distribution of E-vector undulation frequencies) may be different even for the same intensity envelopes. In fact FTS works using slow detectors under the assumption that beams of light containing different optical frequencies are "incoherent" and that is why they do not interfere! In reality, different optical frequencies are coherent and they do "interfere" and produce oscillatory beat or heterodyne currents if the detector and the associated electronics are fast enough. Light beams by themselves are never "incoherent" [16]!

(v) Time-frequency transform, laser mode locking: We will underscore in the next section that well formed light beams do not interact or interfere with (or operate on) each other even when they are physically superposed. Thus, pulse generation by "mode locked" laser is not just due locking the phases of the longitudinal modes of a laser cavity. It is the material dipoles of the saturable absorber or the nonlinear Kerr medium that act as the fast on-off gates to let bursts of energy out of the cavity. So, the ultra short pulse generation community has correctly kept their engineering focus more on the material properties of the gain media, saturable absorber, Kerr medium, etc., rather than on just the phases of the longitudinal modes. Besides, it requires quite a special attention to make a homogeneous gain medium to oscillate in multiple longitudinal modes. We will give simple examples [7, 10, 12].

(vi) Time-frequency transform, pulse dispersion: Pulse dispersion is really time diffraction that people correctly compute by using Maxwell's wave equation and FDTD method for ultra short pulses [30]. Molecules in media always respond very fast to the local amplitude and carrier frequency (-ies). They do not have memory and they cannot wait to determine the Fourier frequencies due to pulses of specific durations and envelopes [7, 20]. Thus, as in classical spectrometry, propagating Fourier transformed frequencies may give "correct" pulse broadening in limiting cases, but that is not the correct physical modeling. We will provide a counter example to make our point clear.

3. Critical view of all the various applications of Fourier transforms

In this section we will be elaborating on all the six different cases of application of Fourier transforms in classical optics that we have just summarized above.

3.1. Space-space transform, optical signal processing. The space-space Fourier transform, or beam propagation from one aperture to a "far-field" aperture, is a direct product of the Huygens-Fresnel Principle and accordingly it is the soundest use of the Fourier theorem. The Huygens-Fresnel (HF) integral always morphs into a

FT-like integral at the plane of convergence of the optical field due to any lens or curved mirror, except for a quadratic phase factor, which would be flat for an "f-f" set up [29].

$$U(P_0) = \frac{-i}{\lambda} \iint_{\Sigma} U(P_1) \frac{\exp(ikr_{01})}{r_{01}} \cos\theta \; ds \tag{1}$$

See diagram of Fig.2a for the one dimensional case of far-field diffraction.

$$U(x) = \frac{e^{ikz} e^{i\frac{kx^2}{2z}}}{i\lambda z} \int_{-\infty}^{+\infty} U(\xi) e^{-i\frac{2\pi\xi x}{\lambda z}} d\xi = C \int_{-\infty}^{+\infty} U(\xi) e^{-i2\pi\xi f_x} d\xi; \;\; f_x = (x/\lambda z) \tag{2}$$

The HF integral for the far-field condition appears essentially identical to a space-space Fourier transform integral. Use of FT in image analysis is equivalent to use of a Huygens-Fresnel Principle. HF-integral prescribes the real physical propagation of light signals. Otherwise this prescription could not have been giving us correct predictions for designing all macro systems like Hubble space telescope and all nano systems like complex nano photonics wave guides (holey fibers, etc.), whether near field or far field cases. Rigorously speaking, temporal evolution cannot be replaced by time-free FT frequencies [20, 27], except under special conditions illustrated in the next section.

$$U(P_0, t) = \frac{-i\nu}{c} \iint_{\Sigma} U(P_1, t) \frac{\exp(i2\pi\nu t)}{r_{01}} \cos\theta \; ds; \;\; t = (r_{01}/c) \tag{3}$$

Engineers carrying out traditional optical signal processing do not require solving this complex space-time coupled integral of Eq.3 because the image plane energy distributions in most optical signal processing are very close to the location of "zero-order" diffraction. HF-secondary wavelets arrive at the "zero-order" zone of the image plane almost simultaneously. However, if one requires considering the intensity distributions around the m-th order diffraction zone, the object illuminating pulse of width δt will be approximately stretched to $\delta t + (m\lambda/c)$. This point is further illustrated for the case of classical spectroscopy where a blazed grating is illuminated by a short pulse and one observes the spectral fringes at the m-th blaze order.

3.2. Delay-frequency transform, Fourier transform spectroscopy (FTS). The application of delay-frequency FT for FTS is an ingenious algorithm proposed and demonstrated by Michelson in late 1800. The "delay" represents the relative time delay introduced between the two arms of the interferometer and the "frequency" represents the actual carrier frequency (E-vector undulations) contained in the light signal. The method is still one of the most extensively used techniques for high resolution infrared spectroscopy. Unlike image processing, this FT technique is not a direct derivative of any major principle of physics. It works for the following set of reasons. First, the recorded fringes are cosine-squared because the electric filed amplitude is of the form of cosine and the recorded optical energy exchange with the filed is quadratic. Second, a passive beam splitter in a Michelson interferometer responds to the light beams of each optical frequency separately, not to the sum of the amplitudes due to different frequencies. Third, the light detectors used to register the fringes must be "time integrating" (averaging) like photographic plates or electronically slow photo current generators. The subtle but very important physics behind these points can be highlighted by urging you to ask the following question. Are light beams with different frequencies "coherent" or "incoherent"?

Let us examine the effects due to two frequencies (modes) of equal amplitudes from a simple CW He-Ne laser through a Fourier transform spectrometer (FTS) and a light beating spectrometer (LBS). Michelson realized that the recorded fringe density $D(\tau)$ by the photographic plate can be expressed as the linear sum of $\cos 2\pi\nu_n\tau$ for each of the actual carrier frequency, without any cross-product between the different frequencies, where $\chi_{(1)}$ is linear susceptibility to dipolar stimulation of the Silver halide molecules facilitating the energy absorption from the EM waves:

$$\begin{aligned} D(\tau) &= \left|\vec{d}(t)\right|^2 \equiv \left|\chi_{(1)} E(t)\right|^2 = \chi_{(1)}^2 \left| e^{i2\pi\nu_1 t} + e^{i2\pi\nu_2 t} + e^{i2\pi\nu_1 (t+\tau)} + e^{i2\pi\nu_2 (t+\tau)} \right|^2 \\ &= \chi_{(1)}^2 \left| e^{i2\pi\nu_1 t} + e^{i2\pi\nu_1 (t+\tau)} \right|^2 + \chi_{(1)}^2 \left| e^{i2\pi\nu_2 t} + e^{i2\pi\nu_2 (t+\tau)} \right|^2 \\ &= \chi_{(1)}^2 4 + 2\chi_{(1)}^2 (\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau) \end{aligned} \tag{4}$$

Separating out the two sets of square modulus terms in the second line of Eq.4 is mathematically correct only under the assumption that neither the beam splitter boundary molecules nor the Silver halide molecules can respond to the cross terms between the different frequencies implied by the first line. This physically correct assumption does not necessarily imply that the EM waves corresponding to different frequencies are automatically "incoherent" to each other. We will come to this again soon [see Eq.6].

Michelson also realized that the scanned undulatory but "DC" fringe density [3rd line of Eq.4] can be converted into an "AC" distribution that can be equated to the sum of the cosines of the fringe density due to each separate frequency with a "conjugate" parameter τ that can be measured by the interferometer and the "delay-frequency" inverse Fourier transform provides the spectrum of the source as in Eq.5. Notice that I have maintained the susceptibility term $\chi_{(1)}$ to underscore that the superposition effects become manifest through detectors' absorption of energy from the superposed EM fields as physical dipoles; detection process is quantum mechanical.

$$\begin{aligned} D_{AC}(\tau) &= 2\chi_{(1)}^2(\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau) \\ \tilde{D}_{AC}(\nu)/2\chi_{(1)}^2 &= \delta(\nu-\nu_1)+\delta(\nu-\nu_2) \end{aligned} \tag{5}$$

This algorithm demonstrated by Michelson works excellent as long as one uses photographic plate or photo detector with time constant much larger than the slowest beat current period (inverse of difference frequency) due to various frequencies in the signal. For faster detectors, we must take square modulus of the sum of all the four terms, as in the first line of the Eq.4, and obtain:

$$\begin{aligned} D(\tau)/\chi_{(1)}^2 &= 4 + 2\cos 2\pi(\nu_1-\nu_2)t \\ &+ 2[\cos 2\pi(\nu_1-\nu_2)(t+\tau) + \cos 2\pi\{(\nu_1-\nu_2)t+\nu_1\tau\} + \cos 2\pi\{(\nu_1-\nu_2)t-\nu_2\tau\}] + 2[\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau] \end{aligned} \tag{6}$$

Carrying out light beating spectroscopy (LBS) with the output from a Michelson interferometer is obviously unwise as it is very complex. We present the Eq.6 only to illustrate our point that analyzing light-matter interaction is critical to understand the measured data due to superposition process. People carry out LBS more easily by directly putting the light beam on to a fast detector, which is a typical undergraduate class demonstration for LBS (or heterodyne spectroscopy). For a given detector, $\chi_{(1)}$ being a constant, we tend to ignore the critical functional roles played by the detecting molecules by paying attention only to the field generated terms as shown below:

$$\begin{aligned} D(\tau)/\chi_{(1)}^2 &= \left|e^{i2\pi\nu_1 t} + e^{i2\pi\nu_2 t}\right|^2 \\ &= 2 + 2\cos 2\pi(\nu_1-\nu_2)t \end{aligned} \tag{7}$$

Obviously, different optical frequencies are not necessarily "incoherent" as we tend to teach in the class. It is the various passive and time-dependent behaviors of light-matter interactions that determine what we can observe and measure in a particular set up. We need to be vigilant not to impose various such joint behaviors to the EM fields alone.

3.3. Time-frequency transform, classical spectrometry

3.3.1. Traditional approach to find the "spectrum" of real signals, which are always time finite? For CW case, the recorded instrumental fringes $I_{cw}(\nu)$ due to complex spectra $S(\nu)$ are expressed as the convolution with the single CW frequency response of a spectrometer $G_{cw}(\nu)$. Hence the real spectra can be extracted by de-convolving the instrumental response:

$$I_{cw}(\nu) = S(\nu) \otimes G_{cw}(\nu) \tag{8}$$

Surprisingly, for a finite pulse, we already assume that we know the "spectrum" by virtue of the Fourier theorem. We express the recorded fringes as a convolution of the Fourier transformed spectral intensity $\left|\tilde{E}(\nu)\right|^2$ with the CW response function of the spectrometer. The recorded fringe pattern is only a validation of our assumption! It does not provide us with any information regarding the actual carrier frequency distribution in the pulse, although, it does contain information about the amplitude envelope of the pulse that we will demonstrate.

$$I_{pls.,cls.}(\nu) = \left|\tilde{E}(\nu)\right|^2 \otimes G_{cw}(\nu) \tag{9}$$

$$\begin{aligned} &\text{Where, signal, } E(t) = a(t)\exp(-i2\pi\nu t), \\ &\text{FT-pair: } E(t) = \int_{-\infty}^{+\infty} \tilde{E}(f)\cdot e^{-i2\pi ft}df \ \ \& \ \tilde{E}(f) = \int_{-\infty}^{+\infty} E(t)e^{+i2\pi ft}dt \end{aligned} \tag{10}$$

Here the Fourier conjugate pair is "time frequency", unlike "delay-frequency" for the case of FTS of the last section. Notice also that we have used f instead of ν to symbolize frequency to deliberately underscore that the mathematical Fourier conjugate variable f is not necessarily the same as the physical carrier frequency that we have dealt with in the last section. The careful readers may pose the classic question that Fourier transform integral represent linear superposition and linear superposition of EM fields is allowed by Maxwell's wave equation. Both are mathematically correct statements. However, mathematically correct and allowed linear superposition cannot override nature’s principle of real physical interactions. Simple physical superposition of well-formed optical beams does not produce re-distribution or re-direction of their energy either in space or in time in the absence of interacting material dipoles [4, 6, 22]. These points will be further clarified through the following analyses.

3.3.2. Reconciling classical and time domain spectrometric analysis. A blazed grating is not a mystified spectrometer. At low intensities it functions as a simple linear pulse replicator (and wave front divider) introducing a train of periodic delay between the replicated pulses. When these pulses are superposed at the focal plane (space-space FT!) of the spectrometer, we register the spectral fringes. As we mentioned earlier, all real signals have finite space and time duration. We are assuming that the entire grating is illuminated by a plane wave of uniform intensity, which may be tough but realizable in practice. The step delay $\tau = m\lambda/c$ is in free space (we are assuming a reflection blaze grating even though the Fig.1 shows it as a transmission grating). N is the total number of slits (steps) of the grating. The mathematical results are summarized below [8, 21].

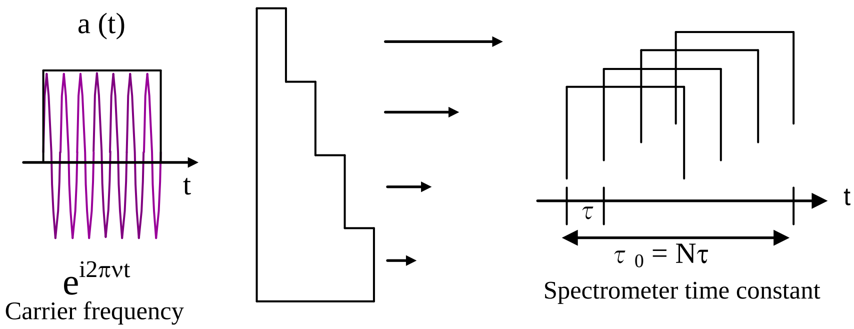


Figure 1. A grating spectrometer behaves as a pulse replicator imposing a periodic train of delays. When this pulse train is physically superposed on a detector array, we measure the spectral fringe distribution [8, 21].

$$\text{Instantaneous fringe intensity: } \left| i_{out}(t) \right|^2 = \left| \sum_{n=0}^{N-1} (1/N)\, a(t-n\tau)\cdot \exp[i2\pi\nu(t-n\tau)] \right|^2 \tag{11}$$

$$\text{Time integrated finge energy or pulse impulse response: } I_{pls}(\nu,\tau)\Big| = \frac{1}{N} + \frac{2}{N^2}\sum_{p=1}^{N-1}(N-p)\gamma(p\tau)\cos[2\pi p\nu\tau] \tag{12}$$

$$\text{Autocorrelation function for all pulse pairs: } \gamma(p\tau) = \int d(t-n\tau)\,d(t-m\tau)\;dt \Big/ \int d^2(t)\;dt \tag{13}$$

Recovering classical CW fringe formula for pulses longer than spectrometer time constant τ_0:

$$\underset{\substack{\delta t \to \tau_0 = N\tau \\ \gamma(p\tau)\to 1}}{Lt.} I_{pls}(\nu,\tau) = \frac{1}{N} + \frac{2}{N^2}\sum_{p=1}^{N-1}(N-p)\cos[2\pi p\nu\tau] \equiv \frac{1}{N^2}\frac{\sin^2 \pi N\nu\tau}{\sin^2 \pi\nu\tau} \equiv I_{cw}(\nu,\tau) \tag{14}$$

Recovering classical spectral fringe broadening for a short pulse with single carrier frequency,

$$\text{using Parseval's energy conservation theorem: } I_{pls}(\nu,\tau) \approx \int_{-\infty}^{\infty} \left| i_{out}(t) \right|^2 dt = I_{cw}(\nu) \otimes \tilde{A}(\nu) \tag{15}$$

This time domain analysis makes it immediately evident that an input pulse of width δt becomes stretched due to “time diffraction” approximately by an amount $\tau_0 = N\tau$ (see Fig.1), which we have mentioned in the earlier section. This should not be confused with material dispersion, c/n(ν), carrier frequency dependent velocity of light. The set of Eq.11-15 represents the most generalized approach to optical spectrometry valid for pulses of any length. Similar expressions for Fabry-Perot spectrometers have also been derived [8, 21, 23]. These equations provide us with a deeper understanding of the physical processes behind spectral information generation. That all spectrometers have a finite characteristic time constant $\tau_0 = N\tau = mN\lambda/c = R\,\lambda/c$ has been missed by books on classical spectrometry, R being the classical resolving power. This is probably because most pulses of light before the days of pulsed lasers were longer than $\tau_0 = R\,\lambda/c$, making Eq.12 & 14 quantitatively equivalent to each other. The other reason of missing out the need for time-domain analysis is evident from Eq.15 which mathematically validates the quantitative data that the time integrated fringe broadening due to a pulse is given by the convolution of the CW fringe pattern with the Fourier intensity spectrum due to the pulse envelope, even though the Fourier frequencies are not real. This can be appreciated two ways. First, the time varying spectral fringe pattern predicted by Eq.11 can be validated by using a streak camera [31]. Obviously, the spectral width does not change with time dictated by the spectrometer time constant. Second, the traditionally accepted time-frequency Fourier bandwidth product is not a fundamental limit of nature. Based on this we have predicted and demonstrated (next section) that spectral super resolution for pulsed light can be achieved two ways. One can de-convolve the pulse-impulse response function, Eq.12, from the time integrated fringe pattern due to a pulse with complex spectrum. Of course, the pulse autocorrelation function must be determined separately. The other approach is to carry out light beating spectroscopy (LBS) by mixing the pulsed signal with a known reference signal, discussed next.

3.3.3. Is it possible to overcome spectrometer resolution limit $\delta\nu\delta t \geq 1$ by LBS? Does an AM signal really contain the Fourier frequencies? If this were true then we would not have needed complex non-linear mathematical formulations and complex optical set up to generate one frequency from another and separately award a Nobel Prize for the field. We have experimentally validated that pure amplitude modulation does not generate new Fourier frequencies and have also validated that spectral super resolution is quite simple to achieve using LBS technique. The experimental set up is shown in Fig.2 and the details have been presented in ref. [8].

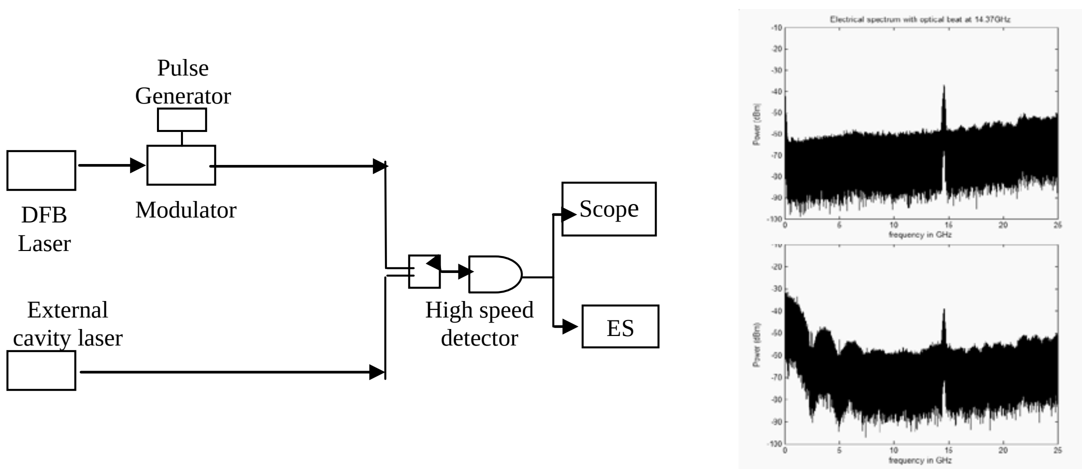


Figure 2. Heterodyne or light beating spectroscopy (LBS) to demonstrate spectral super resolution. *Left*: Experimental set up to combine a DFB laser combined with a tunable CW reference laser. The DFB laser can be run as CW or its amplitude can be modulated externally. *Right*: Heterodyne signals. Top picture shows the photo current analyzed by an electronic spectrum analyzer as a sharp line for the beat (difference) signal when the lasers are CW. The bottom picture shows the same beat line remaining of essentially of same width; but the spectrum due to amplitude modulation is separately displayed at the origin.

Two 1550nm laser beams are mixed on a fast detector and analyzed by an electronic spectrum analyzer (ESA). The DFB laser has a fixed frequency with a line width of 20MHz. The tunable laser has a line width of about 100KHz. The frequency difference was set at 15GHz, which is evident on the top-right picture when both the lasers were maintained CW. Next the DFB laser was modulated with pseudo random super Gaussian (almost rectangular) pulses at 2.5 GHz. The bottom right picture shows that the 15GHz LBS signal has not broadened by any perceptible amount indicating the absence of 2.5 GHz broad Fourier spectrum (note that the vertical scale is logarithmic and the horizontal scale is linear). However, the photo current spectrum as analyzed by the ESA does show the sinc-squared curve of 2.5GHz broadening (first zero) due to 2.5GHz intensity modulated pseudo random square pulses. Since time-frequency Fourier theorem is not a fundamental principle of nature, its corollary, the classic time-frequency bandwidth limitation, cannot also be a fundamental principle of nature.

The photo detector current can be represented by:

$$I(t) = \left| \vec{d}_{cw} e^{-i2\pi\nu_{cw}t} + \vec{d}_p(t) e^{-i2\pi\nu_p t} \right|^2 = d_{cw}^2 + d_p^2(t) + 2\vec{d}_{cw} \cdot \vec{d}_p(t) \cos 2\pi(\nu_{cw} - \nu_p)t \tag{16}$$

The ESA is designed to identify the complex current as various possible harmonics. The first "DC" term d_{cw}^2 on the RHS is neglected by the ESA. The harmonics that can be represented by the second term $d_p^2(t)$ is the sinc-square envelope we see around the origin of the bottom picture of Fig.2b. The harmonic represented by the third term is obviously the narrow peak, $(\nu_{cw} - \nu_p)$ = 15GHz, visible on both the pictures whether the DFB laser is amplitude modulated or not. Using the knowledge of the exact value of the CW tunable source frequency, one can determine the value of the carrier frequency of the amplitude modulated light with a precision that can be many orders of magnitude more precise than we have been taught to believe by the relation $\delta\nu\delta t \geq 1$. Please, note that if complex phase modulation creeps in during the amplitude modulation, the definition of the carrier frequency and the consequent superposition effect induced current will be more complex and a simple LBS interpretation will be difficult.

3.4 Time-frequency transform, coherence theory. In classical or quantum coherence theory we are taught to ignore two important points directly related to the physical process of detecting the effects of superposed signals. The first one has already been mentioned and will be elaborated more in the next section: Well formed light beams do not re-distribute or re-direct their mutual energy distribution (to create interference fringes) in the absence of interacting material dipoles. The second point relates to assigning the degradation of the observed fringe visibility (modulus of autocorrelation, $V(\tau) \sim |\gamma(\tau)|$; see ref.32) due to superposition of unequal amplitudes as the presence of Fourier spectrum of the pulse envelope because the mathematical Autocorrelation (Wiener-Khintchine) theorem appears to explain away this visibility degradation. Alert readers may have already noticed in Fig.1 and Eqns.11-14, that the apparent spectral fringe broadening in a spectrometer due to a short pulse (longer than τ_0) is really due to partial superposition of the displaced pulses (or unequal amplitudes for non-rectangular pulses). We illustrate our point again in Fig.3 for the superposition of two replicated but delayed rectangular pulses produced, say in a Michelson interferometer, from a single incident pulse with a single carrier frequency.

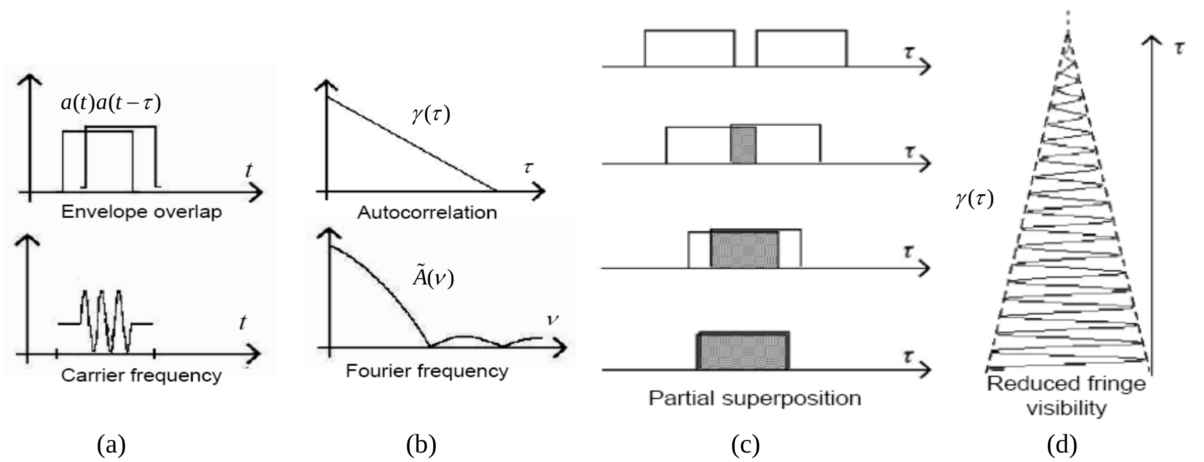


Figure 3. Visibility of fringes can be degraded due to partial overlap of pulses, which does not establish the presence of Fourier frequencies. A streak camera record (right top) could demonstrate the presence of one unique spatial fringe frequency corresponding to the carrier frequency of the pulse with diminishing visibility with relative delay between the superposed replicated pulses. Fourier transform of the fringe visibility (autocorrelation) curve, is not the physical spectrum [16].

The standard mathematics for coherence theory is very simple [32] ignoring the carrier frequency:

$$a(t) = \int \tilde{a}(f) e^{i2\pi ft} \mathrm{d}\nu \;\Leftrightarrow\; \tilde{a}(f) = \int a(t) e^{-i2\pi ft} \mathrm{d}t : \text{ "time-frequency" transform.} \tag{17}$$

$$\text{Fourier frequncy spectrum: } \tilde{A}(f) = |\tilde{a}(f)|^2 \tag{18}$$

$$\gamma(\tau) \equiv \langle a(t)a(t-\tau)\rangle = \int \tilde{A}(f) e^{i2\pi ft} df \;\; : \text{"delay-frequency" transform.} \tag{19}$$

$$\gamma_{\det}(\tau) \equiv \int [\chi_{(1)} a(t)]^* [\chi_{(1)} a(t-\tau)] dt \Big/ \int |\chi_{(1)} a(t)|^2 dt = \int a(t)^* a(t-\tau) dt \Big/ \int |a(t)|^2 dt \equiv \gamma(\tau) \tag{20}$$

We have deliberately used f instead of ν to underscore the difference between mathematical vs. carrier frequencies. Eq.19 is the Autocorrelation (Wiener-Khintchine) theorem. Based on this equation, we claim that the inverse transform of $\gamma(\tau)$ recovers the spectrum $\tilde{A}(f)$, The measured value of $\gamma(\tau)$ can be obtained from either direct pulse autocorrelation measurement or from the interferometric fringe visibility. Also, please, note Eq.20. Whether through fringe visibility or through direct autocorrelation measurements, what we measure is the registered response of some material dipoles with the first-order (in most cases) susceptibility $\chi_{(1)}$ to E-field induced undulations giving rise to energy exchange. So the real autocorrelation should be written as $\gamma_{\det}(\tau)$ to keep us always aware of the quantum mechanical properties of the detector we are using. Unfortunately, the normalized autocorrelation in the linear domain is identical to the field-field autocorrelation, hiding the fact that there is no field-field interaction in almost all cases we consider in routine laboratory set up. The problem would have been recognized much earlier had we discovered some material to measure autocorrelation using its higher order susceptibilities, $\sum_n \vec{\chi}^n_{(1)} \cdot \vec{E}^n(t)$.

There are several mathematical and conceptual problems here. As Fig.3 illustrates, the time integrated fringe visibility degradation is due to partial superposition of delayed pulses, and not due to the presence of real physical Fourier frequencies, even though mathematically it appears that way. The spatial frequency of the fringes under the visibility degrading envelope corresponds to the exact single carrier frequency. Second, $\tilde{A}(f)$ has been derived from "time-frequency" conjugate variable pair. Then we are connecting it with $\gamma(\tau)$ as a Fourier transform pair with conjugate variables "delay-frequency", the running time being integrated away in Eq.17 or Eq.19. Note that τ is a measured parameter and f is a mathematical parameter of convenience. For those attentive readers, if these points are not sufficiently disturbing, let us note that the mathematical proof for the Eq.19 to be rigorously correct, the cross products between the various components Fourier frequency amplitudes of $\tilde{a}(f)$ must be zero [32, 33]! Or, in other words, the very validity of the autocorrelation (Wiener-Khintchine) theorem requires that the Fourier frequencies do not interfere with each other, which opposes the very requirement behind the original time-frequency Fourier theorem! Interestingly, a slow (time integrating) detector, instead of a fast detector, it would average out the beat frequencies due to cross-products between different frequencies. We illustrate this point with the following example [7]. Suppose we have an infinite train of square pulses cut out from a single frequency CW laser [Fig.4, top]. The Fourier intensity spectrum will consist of equally spaced discrete set of "spectral" lines under a sinc-square envelope [Fig.4, bottom left]. The fringe visibility or the autocorrelation measurements will give us repeated triangular function [Fig.4, bottom right]. It is not very difficult to imagine using an inhomogeneously broadened gas laser, like He-Ne laser, to mimic the spectrum of Fig.4 if we start with Gaussian instead of pulses. Then one can interferometrically generate the corresponding visibility (autocorrelation curve) predicted by W-K theorem. The Fig.4 also raises another question. The coherent train of pulses with a single carrier frequency ν_0 can be considered as coming out from a mode locked laser whose intensity spectrum gives Fourier frequencies centered on this carrier frequency ν_0. Does this mean that the so-called transform limited mode lock laser really run in a single carrier frequency? Then, what is the meaning of "mode locked" laser? Let us explore this point further in the next section.

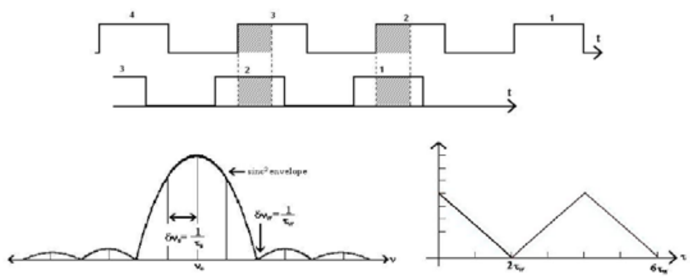


Figure 4. A conceptual demonstration that when a real physical CW physical spectrum mathematically correspond to the Fourier spectrum of an infinite periodic array of pulses with a single carrier frequency, the measured autocorrelation curves are identical even though the two sources have physically different spectrum [**7].**

3.5. Time-frequency transform, laser mode locking. Traditional mode locking theory assumes that in-phase light fields re-arrange "collaboratively" their energy by themselves in time according to the following simple relations:

$$E(\nu_0, t) = \sum_{n=0}^{N-1} e^{i2\pi(\nu_0 + n\delta\nu)t + i\phi_c} \approx \frac{\sin N\pi\delta\nu t}{\sin \pi\delta\nu t} e^{i2\pi\nu_0 t + i\phi_c} \equiv Na(t) e^{i2\pi\nu_0 t + i\phi_c} \tag{21}$$

Corresponding intensity.

$$I(t) = (1/N^2)\left|E(\nu_0, t)\right|^2 = (1/N^2)[\sin^2 N\pi\delta\nu t / \sin^2 \pi\delta\nu t] \tag{22}$$

In reality, saturable absorber simply functions as a time gate! Energy absorbed by the dipoles in the saturable absorber.

$$I_{absorber\ or\ Kerr}(t) = \left|d(t)\right|^2 = (\chi^2_{(1)} / N^2)[\sin^2 N\pi\delta\nu t / \sin^2 \pi\delta\nu t] \tag{23}$$

Can a new mean frequency be generated by the saturable absorber or a Kerr medium out of the cavity longitudinal modes? They are only very fast temporal gates that let out the stored energy from the laser cavity. This assertion can be validated by careful measurements of the tails of the pulses which reflect more the fall and rise time of the "mode-locking" material than the square modulus of the sum of the modes. Fortunately, the laser engineers have been paying careful attention to material response times to keep on advancing the ultra short pulse laser technology.

One can raise further questions. Can the "mode locking" cavity material change the lasing atoms' gain line width from homogeneous to in-homogeneously broadening state for multi-mode oscillation? And then make all the modes operate on each other to create a new mean frequency so the spectral line width becomes "transform limited". By time-domain spectrometric formulation, a "transform limited" fringe corresponds to a single carrier frequency! See Eq.12 that represents the "transformed limited" fringe width. For additional carrier frequencies, the fringe will be broadened further due to their physically displaced intensity distributions in the spectrometer. The results of the detailed spectral analysis of the "mode-locked" pulses from an inhomogeneously broadened He-Ne laser by Allen et al [34] do validate some of our arguments.

If our consistent argument is that the Fourier frequencies are not real, or, not physical to cause real effects in detectors, then what causes the "dispersive" broadening of short pulses? We have already argued that it is the "time diffraction" embedded within the Huygens-Fresnel principle that causes the stretching of the pulses as they diffractively evolve into a sustainable wave packet [see Fig.1]. The point related to dispersion is illustrated with a counter example in the next section. The deeper question of diffractive evolution is addressed after this section.

3.6. Time-frequency transform, pulse dispersion. Let us consider an infinite train of quasi rectangular coherent pulses built out by superposing a finite set of periodic Fourier frequencies [see Fig.5a]. As is customary by our standard dispersion theory, we propagate these Fourier frequencies through different lengths of single mode communication fiber. We have used wavelengths around 1550nm and corresponding values for refractive indices [7, 20]. Notice that the out put pulse train does not monotonically broaden with fiber lengths. After 30km the pulses are broadened and split into a pair; at 600km the main pulses are *narrower* than the input pulses with some energy distributed in the side lobes; and surprisingly, the original pulses are reproduced repeatedly at lengths that are integral multiples of 912 km. This is because all the individual member of the finite set of periodic Fourier frequencies undergoes "modulo-2π" phase delays for multiples of 912km; all the Fourier frequencies arrive in phase at such travel lengths. Autocorrelation measurements and their Fourier transforms at the end of these various lengths would indicate the "spectrum" of the pulse is oscillating back and forth between the original input Fourier frequencies and a broader distribution, which does not make any physical sense!

Pulse stretching due to (diffractive) propagation given by HF principle is different from material dispersion $c/n(\nu)$ experienced by real physical EM wave frequencies. Most of the controversies encountered in pulse broadening measurements related to simple propagation through optical components (lenses, gratings, Fabry-Perots) [8, 21], or for complex phenomena like Slow/Fast light [35], CRDS [36] can be resolved by simply recognizing that mathematical Fourier frequencies do not exist. Even though the Fourier integral represents a linear superposition, the mathematical Fourier frequencies cannot produce the physical effects of interactions with our detecting material dipoles like the actual carrier frequencies can. We "see" light of only real frequencies and only through the "eyes" of material dipoles.

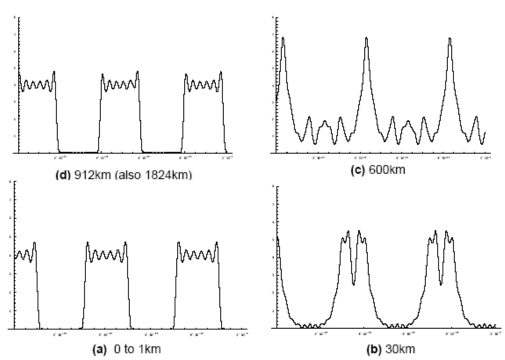


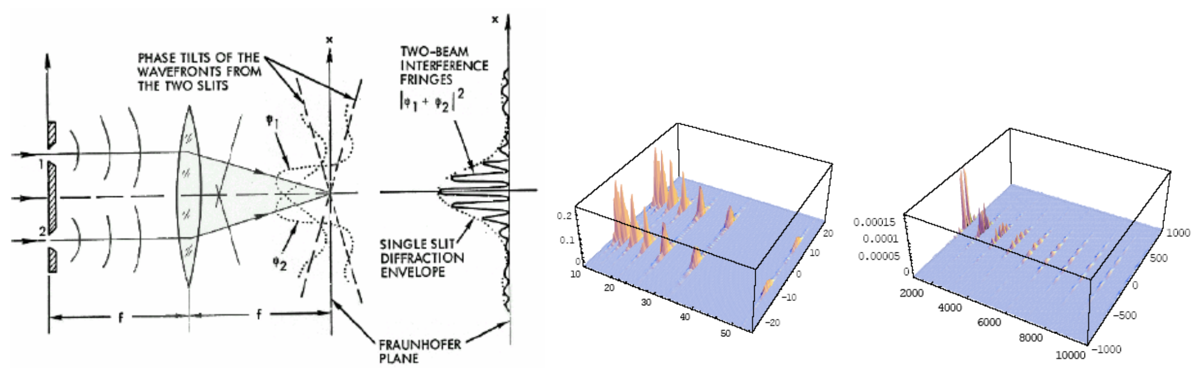
Figure 5. Demonstration of non-physical results of pulse dispersion through different lengths of fibers when mathematical Fourier frequencies, instead of actual carrier frequencies are used. Input pulse - (a). Different odd shapes of output pulses for 30 and 600km fibers – (b) and (c). Restoration of the original pulse for fiber lengths 912km or its integral multiples [7].

4. Recognizing the deep difference between diffraction and interference phenomena

Although in all of our classical optics books, elementary or advanced [32, 37], the phenomena of diffraction and interference are treated in separate chapters, the underlying physical process is considered to be the same superposition effects due to propagating electromagnetic fields themselves. Implication is that the superposed EM waves create the re-distribution or re-direction of energy [13,18,19] by themselves, even though in quantum electrodynamics we consider them to be essentially non-interactive "Bosons", occupying the same space. The reality may appear to be more complex but they are already built into our equations provided we pay close attention to what are periodically undulating whenever we write cos2πνt.

4.1. Diffraction. Huygens-Fresnel principle (HFP) states that the propagation of light wave can be modeled by assuming that every point on the wave acts like a new ***source*** point for a secondary wavelet, sum (superposition) of all of which determines the evolution of the propagating wave. This model, with some modern mathematical "polishing" of the inclination factor [29], works remarkably well from any coarse apertures to very recent complex nano photonic waveguides without failure, whether near fields or far fields. Accepting the mathematical embodiment of HFP as representing physical reality, we have proposed [3, 13] that the propagating EM wave is literally the undulation of a Cosmic Tension Field (CTF) of electromagnetic nature under equilibrium everywhere in space. So, when an undulating material dipole releases its energy and succeeds in disturbing the CTF out of its equilibrium, every point in it tends to push away the imposed disturbance to come back to equilibrium state again. In fact, mathematically and physically this is the model behind the generation of waves on water (under uniform surface tension), waves on musical strings (under mechanical tension) and waves (sound) in air (under pressure tension due to gravity). Thus, Maxwell's wave equation and Huygens-Fresnel diffraction integral complement each other to complete the generation and propagation of EM waves in the vast galactic space containing CTF.

Consider the case of the famous double-slit diffraction pattern introduced by Young over more than two hundred years ago to demonstrate the wave nature of light to over-ride Newton's insistence on "corpuscular" nature of light. The discernible classic cosine double-slit pattern due to superposition of two separate beams from the two well-separated slits can be clearly observed in the far field [see Fig.6 a & b]; the patterns are quite complex in the near fields. The perturbed wave fronts from each of the two individual slits continuously evolve [Fig.1b] by re-grouping themselves into a new collective and sustainable wave form, which becomes a "sinc" pattern in the far-field. Insertion of two orthogonal polarizers over the two slits will destroy the double-slit cosine fringes, but the sinc-square diffraction pattern will remains un-altered. However, the perturbed wavelets from a plane wave by a spatially high frequency grating will re-groups themselves into several sustainable in-phase wave fronts as different diffraction orders [37]. For very high frequency grating these "ordered" beams could emerge out separately even in the "near-field" of a large grating with a beam profile given by $\sin^2 N\phi/\sin^2\phi$, where N is large. Thus diffraction phenomenon is capable of generating energy re-direction (for dense N-slits) and re-distribution (for single slits). The sinusoidal undulations that we write literally represent undulations of the CTF that facilitate the evolution of new sustainable wave group when the original sustainable pattern is seriously perturbed.



(a) **(b)**

Figure 6. Evolution of double slit diffraction patterns from near to the far fields. We are underscoring the difference between diffraction and interference. Insertion of two orthogonal polarizers over the two slits will destroy the superposition ("interference") effects but not he diffraction effects. The physical processes behind diffraction (undulation of the cosmic EM field) and "interference" (superposed EM field induced undulations of the detecting dipoles) are very different [3, 5].

4.2. Interference. In contrast, well formed light beams, even though it continues to spatially expand due to eternal diffractive propagation, they are asymptotically stable with essentially constant angular distribution of the EM field, we call far field distribution. These light beams, even when physically superposed, pass through each other unperturbed [4, 6, 22], without really interfering with each other. Otherwise: (i) The visual world would have been full of spatial and temporal scintillations (speckles). (ii) WDM internet data would have been destroyed by temporal interference (heterodyne effect). (iii) Expanding universe, indicated by Doppler shift, would not have been discernable and measurable. Then why does the superposition of sinusoidal terms work? For pure diffraction the perturbed CTF readjusts its physical undulatory forms providing the emergence of superposition (summation) effect. What facilitates the physical summation effect to emerge when multiple "diffraction free" (sustainable) beams are superposed? It is the undulation of the detecting dipoles, albeit induced by the superposed EM waves, which help manifest the "interference" effects. We see the "interference" effects only when some interacting material dipoles are inserted into the physical domain of superposition, and only when their quantum properties allow them to respond to all the superposed beams simultaneously. Thus, the "visions" of the detectors are restricted by their quantum properties and their communication channels are usually "band limited"; for examples, photo detectors connected with slow electronics are forced to integrate undulatory heterodyne photo current, "telling" us that different optical frequencies are "incoherent" to each other.

In he next two sections we will describe our epistemology for seeking reality in nature.

5. Human logic and supporting mathematics can rarely map cosmic logic in one step

It is of vital importance that we pay careful and very close attention to our epistemology of doing physics. We need to be explicit about the model of our thinking for it determines the model of nature we create. Do we want to understand the actual physical processes behind the incessant cosmic evolution, from femto to macro domains of the universe, which clearly is one continuum? Or, do we want to remain satisfied with mathematical theories that simply produce the measured results without developing adequate physical model to explain the underlying processes?

We start with two sets of examples to underscore that mathematics, embodying human logics, can capture the cosmic logics only partially, because any and all sensors, providing us with observational data, operate through their "vision-limited quantum goggles" and "band-limited communication channels". Pythagoras' equation captures the physical reason, or the geometrical equivalency behind his relation, $c^2 = a^2 + b^2$. The total number of unit squares on the hypotenuse is exactly equal to the sum of those on the other two sides. But I can also provide a mathematical solution for the same problem and quite generally by proposing two linear, instead of one quadratic, relations: $c = 2a - b$ with $\mathrm{a} / b = 4/3$ which reproduces Pythagoras' relation: $c^2 = 4a^2 + b^2 - 4ab = a^2 + b^2$. It just works! But it does not give the elegant "physical picture" of Pythagoras. Very different mathematical logics can arrive at the same prediction! Different mathematical relations giving the same final result implies that we may not be able to determine the unique "cause and effect" relation we are searching for.

If graphical physical pictures were enough, then we may be tempted to revive Ptolemy's *geo-centric* model for our planetary system by introducing only nine "epicycles" and the associated *"free parameters"*, which will be far fewer than used by some of the elegant and "successful" String Theories! We should be intelligent enough to recognize that Nature is a creative system engineer. Accordingly, we should be creative in mind but humble reverse engineers in our approach to *discover* nature's actual realities (processes), rather than *inventing* our own mathematical realities and ask nature to obey them! Technologies derived by emulating nature are not only good for our sustainable evolution, it is also the best way to keep iteratively refining our scientific model of nature. It is clear that we need to develop an epistemology, a systematic model of thinking, which will aid us to overcome the inherent limitations of any and all theories requiring continuous refinement and/or re-modeling.

6. Universality of superposition effects as measurable transformation (SEMT) is at the core of doing science

6.1. Understanding measurement process. Let us first try to understand the process steps behind all of our measurements [9, 38].

(i). Measurable transformation: We can scientifically measure only re-producible quantitative *transformations* (or state change) that is experienced by our interactants (or detector-detectee, or sensor-sensee).

(ii). Energy exchange: Any transformations in any measurable physical parameters in the interactants will always require some *energy exchange* between them.

(iii). Force of interaction: The energy exchange has to be guided by an allowed *force of interaction* between the interactants and it must be strong enough to facilitate the exchange of energy, which are usually constrained by the characteristic limitations (properties) of each interactant.

(iv). Physical superposition: All force rules being distance (range) dependent, energy exchange between the interactants requires that they must experience each other as "locally present" or *physically superposed* entities (experience each other within their sphere of force or influence). Thus, all superposition effects are active "local" processes, not some mysterious game of a passive mathematical principle!

(v). Interactants register and report incomplete information about themselves: All Interactants wear vision-limited "quantum goggles" and report through "band-limited" communication channel. In reality, we are for ever challenged by nature's double blind folds. First, we do not have access to the unified cosmic model, or the complete cosmic puzzle. Second, when we are gathering little puzzle pieces through various experiments, we can rarely gather the complete information about the interactants even for a single experiment. Thus, Gautam Buddha wisely proposed the allegorical story that we are literally born blinds frustratingly trying to describe the cosmic elephant!

6.2. Reality epistemology; iterative cyclic application of CC-LC-(ER)^2 logics. We are trying to solve the gigantic cosmic jig-saw puzzle, but we do not have access to the cosmic picture to validate and/or correct our progress. We now have "solved" over half a dozen of separately little jig-saw puzzles. The corresponding scientific theories are: classical theory, special relativity, general relativity, quantum theory, quantum field theory, cosmology, string theories, and so on. However, for over more than half a century, we have neither unified them into one theory nor discovered any fundamentally new physical principle of nature. So, we need to carefully question our epistemology.

For last several centuries we have been doing physics, first, by accumulating a large number of reproducible observations, which constitute the puzzle pieces. Then we try to assemble a set of them into a solved small puzzle by searching for ***c**onceptual **c**ontinuity* (CC) among the puzzle pieces while trying to enforce ***l**ogical **c**ongruence* (LC) between the set of phenomena we are grouping. This CC-LC epistemology not only constructed the "classical theory" puzzle while keeping focus on discovering realities of nature, it also nurtured the birth of the quantum physics. Unfortunately, quantum theory has shifted the focus more on predicting the outcome of measurements rather than discovering the physical processes behind the interactions whose outcome we measure. Is that a fundamental limit of nature or that of our theory or that of our epistemology? I believe that it is our epistemology that is limiting our discovery of nature's real processes. Accordingly, we are proposing to add two more steps, ER-ER logics, to our classical epistemology of CC-LC.

The first ER stands for ***e**xtracting **r**eality* from successful equations (relations) that are correctly predicting a wide variety of measurements. Our general hypothesis is that if a mathematical relation has succeeded in producing correct predictions, then it must also contain the seeds of reality behind natural processes it is modeling. An excellent example is the classical wave equation for various material media under different kinds of uniform tension in a state of equilibrium that support different kinds of wave propagation. We have extended this analogy in section 4.1 on Huygens-Fresnel principle (HFP) and the corresponding integral. We have argued that if light is really a wave-like phenomenon as predicted by Maxwell's wave equation, then the correct predictive powers of HFP implies that the "empty" cosmic space contains a uniform Comic Tension Field (CTF) of electromagnetic nature whose undulation constitutes light [3, 13]. The validation of this proposition will require time, but we hope the reader can appreciate the implication of the first "ER" epistemology component of the chain CC-LC-ER-ER.

The second "ER" stands for ***e**mergentism and **r**eductionism*. Major parts of classical, quantum and cosmological physics have been built upon the philosophy of reductionism – dissect macro systems into elementary components, understand their properties and then build back the macro system. Unfortunately, we have failed to merge the classical and the quantum physics and their concepts into one harmonious puzzle piece by simply applying CC-LC epistemology. Many scientists [39] and philosophers now appreciating that emergence of complex and rich behavior in macro systems built out of elementary atoms are not always predictable by quantum mechanics even though it models atoms quite accurately. This is where we are proposing that we need to apply iteratively and cyclically *emergentism and reductionism* on complex systems of various sizes, from atoms, molecules and all the way up to galaxies, to synthesize the rich emergent properties by changing and/or modifying some of the accepted properties of the fundamental building blocks.

Just as the children's puzzles are built out of only a small set of basic patterns, the cosmic puzzle is also built out of only a small set of forces, probably four, as we know now. The four forces have allowed us to model the evolving behaviors from elementary particles to clusters of galaxies to a remarkable degree. Such freedom of assembly naturally allows for many "mistaken" fits while solving a complex and large puzzle, especially when we do not have access to the "picture" for the solved puzzle. And, as underscored earlier in the paper, we cannot gather complete information out of any experiment. So, our theories are necessarily built based on incomplete information. Fortunately, human logics (concepts and imaginations) aided by our mathematical logics have been successfully filling the gaps to help us discover the actual cosmic logics. The process is arduous. The purpose of reality epistemology driven by CC-LC-$(ER)^2$ logics is to facilitate the process of merging the solved little puzzles into bigger ones by rejecting and/or re-shuffling some of the individual puzzle pieces. In other words, the proposed epistemology could help modify and/or eliminate some of the individual hypotheses belonging to different theories that may enhance the merger of the theories more harmoniously.

7. Discussions

The key underlying theme is that the Fourier frequencies are not physical and cannot generate any measurable physical effect. The author hopes that optical design engineers would appreciate that a "perfectly" working mathematical theory behind a particular optical instrument does not pre-empt all further innovations in that particular sub-field. We have shown that even the most powerful two century old Fourier theorem, while providing "perfectly" working mathematical formulations have various limitations simply because we have neglected to pay attention to the physical processes behind light-matter interactions. We have given specific examples where new innovations will be feasible. New spectrometers can be designed with spectral super resolution with orders of magnitude more precision than the currently believed limit of $\delta\nu\delta t \geq 1$. The focus of this paper has been the strengths and weaknesses behind applications of Fourier theorem in classical optics. If the reality of mathematical Fourier frequencies becomes questionable through such an extensive number of classical optical experiments, then it will also be an important question for quantum optics. A separate paper will address the applications of Fourier theorem in quantum optics. For example, the photon in quantum optics is defined as a Fourier mode of the "vacuum". Is this a valid description for a photon? Should photon be defined as a classical wave packet, which after generation, evolves out according to the prescription provided by the Huygens-Fresnel principle [5, 9, 11]? After all HFP has never been invalidated.

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