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WHAT IS COHERENCE?

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Abstract: Superposition fringes are produced by detector arrays as photo chemical decomposition or photo electron emission, rather than through pure field-field correlation. So the observed degradation in fringe visibility reflects detectors' quantum properties and the relevant integration time constants along with the parameters of the superposed fields. We present improved interpretation and classification of coherence function, or fringe visibility, based on this light-matter interaction process.

1. INTRODUCTION

Coherence in reality is a complex set of quantum mechanical responses of detecting dipoles due to simultaneous stimulations induced by multiple superposed EM waves. Coherence is not just the correlation between two electromagnetic fields, even though it is the fields that initiate the superposition effects that we want to investigate and measure [1]. What we observe is a restricted set of information generated in the measuring instrument as some transformations experienced by the clusters of detecting molecules due to light induced stimulations and followed by energy transfer. The information that we gather from our measurements about light-matter interactions is fundamentally limited for two reasons. First, because the detecting molecules wear "quantum goggles" of limited vision as they can respond to light possessing only a specific band of frequencies. The detectors ignore the out-of-band frequencies. Second, because the products of transformation, photo electron charges, need to be extracted as a measurable current through a complex set of circuit and a meter or a sophisticated scope. Such measuring systems are inherently "band limited" information conduits because they modify and/or distort the information embedded in the released original photo electron charge due to the finite LCR-time constants of the circuits. Similar arguments apply to photographic plates also. Ag-halide molecules are quantum mechanical devices and their photo chemical dissociation requires a radiation of some minimum optical frequency. Then the photographic plates have to undergo complex multi-step chemical processing before the original "exposure" becomes manifest as variable darkness. Such photographic fringes cannot provide us with all the original information brought and delivered on to the detecting molecules by the superposed optical fields accurately.

Let us now identify a few contradictions and paradoxes built into our current coherence theory as it is built upon field-field correlation rather than upon detection processes that make the measured fringes appear in our instruments. First, we say that white light from thermal sources are completely incoherent. But we know how to generate white light fringes around the m=0 order location in an interferometer [$m = \Delta / \lambda$]. Second, Michelson's Fourier transform spectrometry [FTS] works based on the hypothesis that different frequencies do not interfere. This implies that they are incoherent with each other. But, after the discovery of high-speed detectors, we learnt that different frequencies do generate superposition effects and give rise to beat frequencies. Third, we assume and experimentally observe that orthogonally polarized light beams do not give interference fringes implying that they are incoherent to each other. Yet, we add orthogonally polarized light beams with $\pi / 2$ relative phase delay and claim that the beams sum themselves to produce elliptically or circularly rotating electric vector. Fourth, inspite of absence of any temporal variations in a CW laser beam, we call it the "variation of *temporal* coherence" when the visibility variation is produced due to the presence of multiple frequencies in the beam. These paradoxical interpretations of the degree of coherence are the result of historical development of the field of coherence in discrete steps. It is now necessary for us to reorganize these interpretations into a conceptually coherent model by imposing logical congruence among them based on improved understandings of the various physical interaction processes that give rise to fringe visibility that we measure.

2. TEMPORAL, SPECTRAL AND SPATIAL FRINGE VISIBILITY

The expressions for the degree of coherence or the autocorrelation between two superposed fields and its normalized expression can be presented as [2]:

$$\Gamma_{1,2}(\tau) = \left\langle a_1^*(t) a_2(t-\tau) \right\rangle; \quad \Gamma_{p,p} = \left\langle a_p^*(t) a_p(t) \right\rangle \equiv I_p \qquad (1a)$$

$$\gamma(\tau) = \frac{\left\langle a_1^*(t) a_2(t-\tau) \right\rangle}{\left\langle |a_1(t)|^2 \right\rangle^{1/2} \left\langle |a_2(t-\tau)|^2 \right\rangle^{1/2}} = \left\langle a_1^*(t) a_2(t-\tau) \right\rangle \Big/ \sqrt{I_1}\sqrt{I_2} \qquad (1b)$$

For an ergodic system, the ensemble average $\langle \ \rangle$ can be replaced by time integration:

$$\gamma(\tau) = \int_0^{t>2\delta t} a(t) a(t-\tau) dt \Big/ \int_0^{t>2\delta t} a^2(t) dt \qquad (1c)$$

If the half width of $a(t)$ is δt then the integration time can be terminated after $2\delta t$ without loss of generality. Let us also note that photo electron emission is a quantum mechanical process that can be measured only through ensemble averaging, which is a very successful prescription of QM. Measurements should be compared with $< \psi^* \psi >$ and not the single event $\psi^* \psi$. Detection of a single quantum event does not help us decisively validate the QM predictions regarding superposition fringes [3, 4].

Let us now look at the autocorrelation (Wiener-Khintchine or W-K) theorem [2]. It states that the normalized autocorrelation function and the normalized spectral intensity function $|\tilde{a}(\nu)|^2$ form a Fourier transform pair. Note that the pair of conjugate variables for the Fourier transform is frequency and delay (ν, τ), both being physical parameters in real experiments:

$$\gamma_\nu(\tau) = \int_0^\infty |\tilde{a}(\nu)|^2_{norm} e^{-i2\pi\nu\tau} d\tau; \quad |\tilde{a}(\nu)|^2_{norm} = \int_0^\infty \gamma_\nu(\tau) e^{i2\pi\nu\tau} d\tau \qquad (2a)$$

We believe that $\gamma_\nu(\tau)$ in Eq.2a should be interpreted as representing *spectral visibility* or *coherence*, in contrast to traditional *temporal coherence* function, when the signal $|\tilde{a}(\nu)|^2$ represents real physical CW spectral intensity distribution. Anybody who has done some un-equal path interferometry (or holography) with a CW multiline (frequency) He-Ne laser has observed the oscillatory nature of $\gamma_\nu(\tau)$. *Temporal visibility or coherence* interpretation should be reserved for those situations when the signal is literally of finite time duration. For example, if we chop out a pulse $a(t)\exp[-i2\pi\nu t]$ from a CW single frequency laser, or clip out a single pulse $a(t)\exp[-i2\pi\nu t]$ from a transform limited perfectly mode locked laser pulse train [1], *the time integrated* interferogram from an un-equal path interferometer will show the measured coherence function Eq.2b that is very much like that of Eq.2a. However, if one records the fringes with streak camera having a temporal resolution at least an order of magnitude faster than the width of the pulse $a(t)$, the fringe visibility or the correlation function will evolve with time, which when integrated will again reproduce Eq.2b. Thus the integration time of the detector dictates the observed results for pulsed light.

$$\gamma_t(\tau) = \int_0^\infty |\tilde{a}(f)|^2_{norm} e^{-i2\pi\nu\tau} d\tau; \quad |\tilde{a}(f)|^2_{norm} = \int_0^\infty \gamma_t(\tau) e^{i2\pi\nu\tau} d\tau \qquad (2b)$$

Where,

$$a(t) = \int_0^\infty \tilde{a}(f) e^{-i2\pi ft} df, \quad \tilde{a}(f) = \int_0^\infty a(t) e^{i2\pi ft} dt \qquad (3)$$

We make the traditional interpretation of the signal $a(t)\exp[-i2\pi\nu t]$: $a(t)$ is an imagined temporal envelope function describing the amplitude variation of the electric vector oscillating at the unique carrier frequency ν. Now we want to underscore the difference between $\tilde{a}(f)$ and $\tilde{a}(\nu)$ and hence between $\gamma_t(\tau)$ and $\gamma_\nu(\tau)$. First, let us recognize that the detected fringe energy variation is due to the resultant phases of the two superposed beams as experienced by the detector. The real physical parameters that cause the phase variation of an electric vector are its frequency $2\pi\nu t$ and the propagational delay $(2\pi\nu t + \tau)$. So, it is logical to have two distinct visibility functions, $\gamma_t(\tau)$, $\gamma_\nu(\tau)$. The frequency ν in $\tilde{a}(\nu)$ represents real physical CW frequency distribution of the source. We have used the "tilda" symbol on $\tilde{a}(\nu)$ only to maintain mathematical symmetry that helps us to bring out the contradiction buried in the W-K theorem.

In contrast, the frequency distribution f in $\tilde{a}(f)$ is fictitious mathematical frequency that does not have real existence, even though it matches up with measured data under time-integrated situations. It is important to note a critical mathematical assumption in the derivation of Eq.2b using Eq.3. One must assume that the superposition or the cross terms generated by $\left|\tilde{a}(f)\right|^2_{norm}$ under the integral of Eq.2b vanish, or they do not interfere. Such an assumption is correct only for slow, time integrating detectors. Fast detectors will routinely detect heterodyne beat signals due to the cross terms that are neglected while deriving Eq.2b and the assumption is invalid. If an optical signal is more complex and contains multiple carrier frequencies under the same time envelope, the measured fringe visibility will be a complex function of both the *temporal and spectral visibility* functions $\gamma_t(\tau)$ and $\gamma_\nu(\tau)$.

Readers should note that the assumption of non-interference of different frequencies lies at the heart of the success of Michelson's Fourier transform spectrometry (FTS) embedded in Eq.2b since FTS always use slow detectors. Treating $\tilde{a}(f)$ and $\tilde{a}(\nu)$ as identical implies that the Fourier's theorem [Eq.3] is a principle of physics; however, we have never formally declared so. Mathematical correctness of Fourier's linear superposition theorem cannot over ride the necessity of physical interaction process between real interactants that make the superposition effects become manifest (measurable transformation). Generation new optical frequencies always require non-linear susceptibilities ${}^{(n)}\chi E^n$ of some material dipoles to come into active play.

Spatial visibility or *coherence* refers to correlation between the phases at a set of spatially distinct points of a wave front. Such a correlation factor can be less than unity (spatially incoherent or partially coherent) only if the wave front is composed of radiation from more than one independent (phase uncorrelated) sources. *Spatial visibility* has the unique property that $\gamma_x(\delta x)$ evolves towards unity for a pair of points of given separation $\delta x \equiv (x_2 - x_1)$ as the propagation distance increases from the source due to diffractive propagation, which has been formalized by the van Cittert-Zernike theorem [2]:

$$\begin{aligned}\gamma_x(\delta x) &= \frac{\left\langle a_1^*(x_1)a_2(x_2)\right\rangle}{\left\langle\left|a_1(x_1)\right|^2\right\rangle^{1/2}\left\langle\left|a_2(x_2)\right|^2\right\rangle^{1/2}} \\ &= \left\langle a_1^*(x_1)a_2(x_2)\right\rangle \Big/ \sqrt{I_1}\sqrt{I_2}\end{aligned} \qquad (4)$$

Thus the degree of *spatial visibility* $\gamma_x(\delta x)$ is distinctly different from the *temporal visibility* $\gamma_t(\tau)$ and *spectral visibility* $\gamma_\nu(\tau)$. The latter two degrees of coherence do not evolve with free space propagation of the light beams, especially when the diffractive evolution of the wave fronts are negligible, unless they propagate through optical devices that can modify the temporal and spectral characteristics of the beams.

3. PHYSICAL ORIGIN OF AUTOCORRELATION EXPRESSION

If $\gamma(\tau)$ is to be a real physical observable, then all the coherence related relations shown above must represent some real physical interaction process. Let us consider a simple experimental case from where such a relation may arise naturally. A two-beam interferometer creates superposition of two signals $a_1(t)\exp[i2\pi\nu t]$ and $a_2(t-\tau)\exp[i2\pi\nu(t-\tau)]$ at the output. It is customary to represent the detected intensity:

$$\begin{aligned}I(\tau) &= \left\langle\left|a_1(t)e^{i2\pi\nu t} + a_2(t-\tau)e^{i2\pi\nu(t-\tau)}\right|^2\right\rangle \\ &= \left\langle\left|a_1\right|^2\right\rangle + \left\langle\left|a_2\right|^2\right\rangle + 2\left\langle a^*_1(t)a_2(t-\tau)\right\rangle\cos 2\pi\nu\tau \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\frac{\left\langle a^*_1(t)a_2(t-\tau)\right\rangle}{\sqrt{I_1}\sqrt{I_2}}\cos 2\pi\nu\tau \\ &= (I_1+I_2)[1+\frac{2\sqrt{I_1}\sqrt{I_2}}{(I_1+I_2)}\;\gamma_t(\tau)\cos 2\pi\nu\tau] \\ &= (I_1+I_2)[1+\beta\gamma_t(\tau)\cos 2\pi\nu\tau];\quad \beta \equiv \frac{2\sqrt{I_1}\sqrt{I_2}}{(I_1+I_2)}\end{aligned} \qquad (5)$$

Michelson defined visibility as:

$$\begin{aligned}V(\tau) &\equiv \frac{I_{\max}(\tau) - I_{\min}(\tau)}{I_{\max}(\tau) + I_{\min}(\tau)} = \beta\gamma(\tau) \\ &= \gamma(\tau)\ [\beta = 1 \text{ for } I_1 = I_2]\end{aligned} \qquad (6)$$

The first line of Eq.5 implies that the two EM waves have interfered by themselves and produced the fringes by re-distributing (re-directing) their energy, which is not correct. Our classical observations support this claim of non-interference between well formed light beams [5]. Our eyes or cameras can receive a beam from a scenery and construct a clearly stable and unchanging image even though this beam is routinely crossed by innumerable other light beams traveling in every possible directions. So the superposition effect implied by the "+"operator in Eq.5 is not valid since the two beams a_1 and a_2 cannot operate (interact) with each other by themselves. It is the dipole molecules of the detector that respond to both the superposed fields simultaneously and sum the complex stimulations before absorbing quantum mechanically allowed amount of energy from both the fields. If the incident field is $E_{1,2}(t) = a_{1,2}\,\mathrm{e}^{i2\pi\nu t}$, then the dipole stimulation is:

$$\psi_{1,2}(t) = {}^{(1)}\chi a_{1,2}\,\mathrm{e}^{i2\pi\nu t} \qquad (7)$$

Where ${}^{(1)}\chi$ is the linear (first order) susceptibility of the molecule to polarization by the EM wave. So Eq.5 needs to be re-written as summation of dipole stimulations underscoring the ongoing physical process before energy can be absorbed.

$$\Psi(\tau) = {}^{(1)}\chi a_1(t)\,\mathrm{e}^{i2\pi\nu t} + {}^{(1)}\chi a_2(t-\tau)\,\mathrm{e}^{i2\pi\nu(t-\tau)} \quad (8)$$

The rate of photo electron emission (energy absorbed), $\Psi^*\Psi$ is now given by:

$$\left\langle \left|\Psi(\tau)\right|^2 \right\rangle = \left\langle \left| {}^{(1)}\chi a_1\,\mathrm{e}^{i2\pi\nu t} + {}^{(1)}\chi a_2\,\mathrm{e}^{i2\pi\nu(t-\tau)} \right|^2 \right\rangle \quad (9)$$
$$= {}^{(1)}\chi^2 (I_1 + I_2)[1 + \beta\gamma_t(\tau)\cos 2\pi\nu\tau]$$

Because ${}^{(1)}\chi$ is a common constant, comparison of Eq.5 & 9 tell us that the mathematical expressions for the degree of coherence $\gamma_t(\tau)$ and the visibility $V(\tau)$, as shown in Eq.6, remains unaltered. One can now legitimately raise the question: Is it worth debating superiority one mathematical equations [Eq.5] over another (Eq.9), if both of them generate identical expressions for some measurable quantities? Our answer is affirmative because Eq.5, in spite of predicting correct measured value (within a constant) it implies a wrong physical assumption – light beams interfere with each other even in the absence of detecting molecules. It is generally understood that when an equations works in predicting real measurements, it has captured some reality (physical process of interaction) in nature. But this "process" has to be articulated by human imaginations (visualization of the invisible processes). The approach to such visualization is to meticulously associate each algebraic symbol with some actual physical parameter of the interactants being modeled or the transformed state connected by the "equal" sign. We also need to be sure that the mathematical operators depict the right force of interaction that is at the root of transformation we trying to model [3]. The significance of referring to the physical process carried out by the detecting dipoles was mentioned in section 2 in relation to FTS that works only when we use time integrating slow detector under the assumption of non-interference of light of different frequencies, which is also correctly modeled by W-K theorem. Unfortunately, W-K theorem and FTS procedure does not work when we use a very fast detector!

DISCUSSIONS

Well formed light beams do not interfere or interact with each other. Superposition fringes are produced by the detecting dipoles and hence the time constants of detecting systems determine the outcomes in the measured visibility. Attention to the light-matter interaction processes helps us understand the physical meaning behind the degradation of the fringe visibility. Accordingly, we have classified three categories of fringe visibility that are descriptive of three distinctly different characteristics of light beams – *temporal visibility* (pulsed light), *spectral visibility* (multi frequency light) and *spatial visibility* (independent multiple emitters).

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