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Generalized quantitative approach to two-beam fringe visibility (coherence) with different polarizations and frequencies

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Abstract

All two-beam interferometry eventually reduces to quantitative measurement of the effective fringe visibility (or degree of coherence) in some form. We present generalized analytical and experimental results of visibility for the cases of two beam Poynting vectors both collinear (scanning fringe mode) and non-collinear (spatial fringe mode) with different polarizations and frequencies. This leads to a much broader and deeper understanding of the roles of material dipoles (beam splitters & detectors; both classical and quantum) in measured coherence effects that are not explicitly addressed in the traditional coherence theory. Coherence theory should be presented as correlation between sensing dipole undulations that are simultaneously induced by superposed light beams rather than as correlation between the optical fields. This generalized understanding of the physical processes behind coherence phenomenon will open up (i) better understanding of the nature of light and (ii) many more innovative approaches to quantitative interferometry.

Key words: Coherence, degree of coherence, correlation function, interferometry; superposition principle, locality of interference fringes, interference of polarized light, interference of light of different frequencies.

1. INTRODUCTION

We "see" the presence of light energy only through the "eyes" of material dipoles [1]. So the key theme of this paper is to establish that the optical coherence theory should be presented as correlation between the simultaneous dipole stimulations induced by the simultaneously present (superposed) light fields. We get deprived of visualizing the light-matter interaction processes, if we present "coherence" only as the correlation between the optical fields. We also believe that this is one of the central reasons behind plausible interpretation of optical interference as "non-local". We should faithfully apply all that is demanded by our "successful" theories to appreciate the roots of their successes and their limitations towards further advancement of our theories. Quantum Electrodynamics (QED) claims photons to be Bosons implying that they can occupy the same physical space without interacting with each other. An obvious example is the recoverability of a laser beam unperturbed after sending it through a pin hole with a focusing lens before it. It is the Classical Electrodynamics (CED) that has never formally recognized non-interference between well formed light beams even though it is obvious from our daily observations. Images we form on our retina by intercepting a light beam from a sight of our interest generally remains stable even though the beam comes to us after crossing through innumerable other light beams propagating in different directions.

Well formed optical beams do not interact with each other to modify or re-distribution their field energy distributions [1]. This is obvious from the fact that two intersecting laser beams remain unperturbed after their crossing region unless we insert some interacting material medium within their crossing volume. Superposition (interference) fringes never become manifest without the aid of interacting material dipoles that are allowed to interact simultaneously with all the beams superposed on them. So, optical coherence should be presented as correlation of dipoles stimulations induced by the fields. However, the current coherence theory, developed during the 1st half of the 20th century and formalized during the 2nd half of the 20th century, including its quantum mechanical version, has been serving us well that is supported by a wide variety of experimental measurements. So, the current mathematical theory must have captured the essential aspects of light-matter interaction processes, even though the present coherence theory is formulated as field-field correlation. This is simply because the mathematical process of normalization of the correlation function eliminates the light-matter interaction parameter, the linear susceptibility factor ${}^{(1)}\vec{\chi}$ to polarized dipole undulation induced by the electric vector $\vec{E}$ of the light wave.

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What are the scientific and engineering utilities of raising such a "trivial" issue, a normalizing factor? Refocusing our attention to this susceptibility factor $^{(1)}\vec{\chi}$ will re-direct our attention in doing proper physics – trying to understand and visualize the invisible light-matter interaction processes. Since engineering innovations derives from emulating physical processes allowed by nature and its rules, a deeper understanding of the physical processes behind light-matter interaction would open up the possibility of many more practical innovations, especially, in light of the emergence of nano photonics where the nano matter clusters are straddling between classical quantum domains. For fundamental physics, several quantum mechanical interpretations become questionable. How can superposition fringes be formed "non-locally" when sub-nanometric detecting molecules must first experience all the superposed light waves on them before they can generate the superposition (interference) fringe patterns? Could it be correct that only the absence of knowledge (information) by human experimenters as to "which way light (photons) travel" is at the root of generation of fringes?

CED has not succeeded in defining any force that can mediate direct interactions between well-formed light beams for our routine laboratory experiments. And QED calculation demands almost un-attainable high intensity to make pure photon-photon interactions observable unaided by any material medium. So, we can safely conclude that well-formed light beams do not interfere with each other in the absence of interacting material media.

We "see" light only through our subjective interpretation by our cerebral neural net of the information sent to it through several intermediate transformations triggered by the original transformation induced on our retinal molecules. Neither the retinal molecules can respond to all the key parameters of the light beams nor can they transfer all the detailed transformations experienced by them all the way to the macro observation (interpretation) system. Photons are not painted with different colors by the creator and yet we see vivid colors all around us; the brain is a marvelous interpreter for our survival!

Such observational limitations are scientifically applicable to all of our light sensors connected to reading instruments. All information about light-matter interactions and the follow-on measurable transformations is gathered indirectly through "band limited" macro instruments. Further, the registered transformation is based on the involvement of a limited number of parameters of the light beam and the sensing material dipoles. Thus, even the interactions are "band-limited" and vary from sensor to sensor. We do not yet know the true nature of light [2,3]!

Can we know the nature of light any better? If we focus our attention to understand the real physical processes behind light-matter interactions and consistently try to visualize the invisible processes using our evolving theories, we will be able to gather more and more information about the true nature of light even within the current bounds of CED and QED. And, eventually, we should be able to formulate the next theory that is better than the current QED. Is "non-interference of light" a scientifically useful hypothesis for further exploration of the nature of light [3, see ch.6 of 2]?

The purpose of this article is to draw upon the previous publications [1,3,4] on this concept and generalize this hypothesis to "coherent" light beams of different polarizations and different frequencies and present a consistent methodology to compute and measure correlations (fringe visibility) for superposed light beams based on responses of material sensors. We will demonstrate that our approach brings better and more coherent understanding of the light-matter interactions and hence the nature of light.

This paper is being presented in the section "On the Fringe" appropriately implying that the contents of this paper do not belong to the mainstream thinking. Accordingly, we appreciate the acceptance of this paper by the SPIE organizing committee members. This is especially heartening for one of the authors (CR) since his papers on the same basic theme of "coherence" have been rejected three times earlier by two major international optics conferences.

2. COHERENCE THEORY AS WE KNOW IT

From complex analytical signal theory, it has been established [5] that Michelson's fringe visibility $\mathrm{V}(\tau)$ in a two-beam interferometer is the same as the modulus of the normalized autocorrelation function $\gamma(\tau)$. The interferometer generates two relatively delayed signal envelopes, $a(t)$ and $a(t-\tau)$, from the same original signal $a(t)\exp[-i2\pi\nu t]$ with an optical carrier frequency ν:

$$\left|\gamma(\tau)\right| = V(\tau) \equiv (< I_{\max} > - < I_{\min} >)/(< I_{\max} > + < I_{\min} >) \tag{1}$$

The normalized degree of coherence or the autocorrelation between two superposed fields is presented as:

$$\gamma(\tau) = \frac{< a^*(t)a(t-\tau) >}{< a^*(t)a(t) >} = \frac{\int_0^\infty a^*(t)a(t-\tau)dt}{\int_0^\infty \left|a(t)\right|^2 dt} \tag{2}$$

Then one can derive the autocorrelation (or Wiener-Khintchine) theorem. It states that the normalized autocorrelation function and the normalized Fourier intensity spectrum form a Fourier transform pair. Note that the pair of conjugate variables for the Fourier transform is frequency and delay (ν,τ), both being physical parameters in real experiments:

$$\gamma(\tau)=\int_0^\infty \left|\tilde{a}(\nu)\right|^2_{norm} e^{-i2\pi\nu\tau}d\tau \quad \text{and} \quad \left|\tilde{a}(\nu)\right|^2_{norm}=\int_0^\infty \gamma(\tau)e^{i2\pi\nu\tau}d\tau \tag{3}$$

Eq.2 represents actual *temporal coherence* due to relative delay τ forcing superposition of unequal amplitudes and hence reduced visibility; there are no other optical frequencies present in the original signal $a(t)\exp[-i2\pi\nu t]$. Eq.3 represents *spectral coherence* provided $\left|\tilde{a}(\nu)\right|^2_{norm}$ is the actual physical spectrum of the signal we are using and it is CW. But we have abandoned the difference between the *temporal coherence* due to a time finite signal with a single carrier frequency and the *spectral coherence* due to CW signal with true frequency spread originated at the source. This is due to identification of the actual carrier frequencies ν of $\tilde{a}(\nu)$ with the Fourier's mathematical frequencies f of $\tilde{a}(f)$ because we need to use the original Fourier theorem [Eq.4] to derive the autocorrelation theorem of Eq.3.

$$a(t)=\int_0^\infty \tilde{a}(f)e^{-i2\pi ft}df, \text{ where } \tilde{a}(f)=\int_0^\infty a(t)e^{i2\pi ft}dt \tag{4}$$

Some how we have been ignoring the fact that the derivation of Eq.3 using Eq.4 requires the assumption that the cross terms of $\left|\tilde{a}(\nu)\right|^2_{norm}$ are zero, or there are no "interference" between different optical frequencies. In the optical domain this assumption is correct only as long as we use slow detectors and electronics that are incapable of responding to the heterodyne beat current produced in the photo detector [3]. In fact, this assumption of non-interference of different frequencies lies at the heart of success of Michelson's Fourier transform spectrometry grounded on Eq.3. Treating $\tilde{a}(f)$ and $\tilde{a}(\nu)$ as identical implies that the Fourier's theorem [Eq.4] is a principle of physics; however, we have not formally declared so. Correctness of a particular mathematical linear superposition theorem cannot over ride the necessity of real physical interaction process in nature, which can generate new optical frequencies from an incident single carrier frequency. Another note of interest is that *spatial coherence* in general is a manifestation of a mixture of *spectral and temporal coherence* for the most generalized signal like spontaneous emission consisting of many pulses with many different carrier frequencies.

In the following sections we devote our attention to the roles played by material media in light-matter interactions to register superposition effects due to two beams of "coherent" light. "Coherent" in the sense that each light pulse $a(t)\exp[-i2\pi\nu t]$ has duration long enough (number of cycles) for the interacting materials either to absorb from or re-direct energies out of the incident light beams. But the observed fringe visibility will be dictated by the properties of the interacting material dipoles.

3. ALL OPTICAL TWO-BEAM SUPERPOSITION INTERFEROMETERS REQUIRE UNDERSTANDING BEAM SPLITTER FUNCTION

Consider the final beam splitter BS2 of a Mach-Zehnder interferometer (Fig.1) shown enlarged on the right hand side of the diagram under two different conditions – (i) the Poynting vectors are collinear when the interferometer is in the scanning fringe mode and (ii) the Poynting vectors are non-collinear when the interferometer is in the spatial fringe mode. Obviously, in the absence of the beam splitter BS2, the two well defined coherent beams, although intersecting each other within a finite volume, will emerge out unchanged without any "memory" of the experience of their temporary superposition. This is what we mean by non-interference of well formed light beams. In the presence of the beam splitters, when the phases of the two beams are given the right value and the reflection and transmission conditions for the two beams are such that their Poynting vectors are collinear, a 50% beam splitter can act as a 100% transmitter or a 100% reflector. We know that such superposition (interference) effect cannot become manifest without the active participation of the boundary molecules of the beam splitter and without the simultaneous presence of both the beams. Obviously, the sum of the energies in the two directions must be conserved. Let us re-discover the necessary condition. The incident beams are generated from the same laser with single frequency and parallel polarization with a relative phase delay τ. Then the complex dipole stimulation amplitudes are, assuming $d_x={}^{(1)}\chi a_x$:

$$\begin{aligned} d_{right}(\tau)&=rd_1e^{i2\pi\nu(t+\tau)}+td_2e^{i2\pi\nu t}\\ d_{up}(\tau)&=td_1e^{i2\pi\nu(t+\tau)}+rd_2e^{i2\pi\nu t}\end{aligned} \tag{6}$$

The corresponding registered intensities are:

$$D_{\text{right}}(\tau) = \left|d_{\text{right}}\right|^2 = r^2 d_1^{\ 2} + t^2 d_2^{\ 2} + 2trd_1 d_2 \cos 2\pi\nu\tau$$
$$D_{\text{up}}(\tau) = \left|d_{up}\right|^2 = t^2 d_1^{\ 2} + r^2 d_2^{\ 2} + tr2d_1 d_2 \cos 2\pi\nu\tau \tag{7}$$

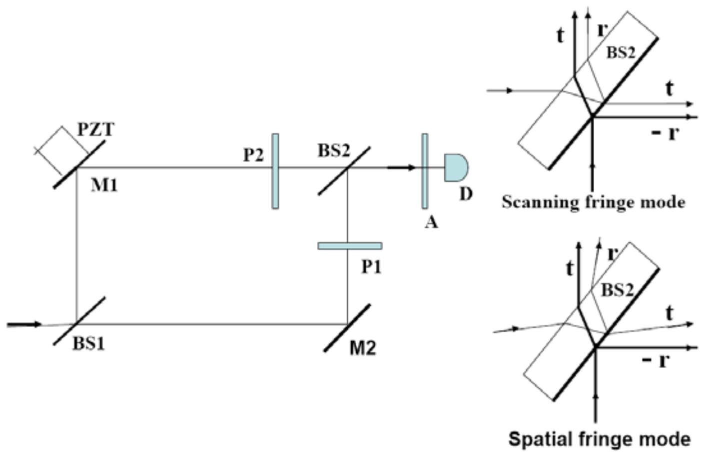

Figure 1. A typical Mach-Zehnder interferometer with polarizers in two of its arms. The final beam splitter is shown enlarged under two conditions – the Poynting vectors are collinear (right top) and non-collinear (right bottom). For convenience of depicting the different properties experienced by the beams, the representative rays are slightly physically displaced.

Conservation of energy tells us that $t^2 + r^2 = 1$. But, here we require that the sum of D_{right} and D_{up} must remain constant and equal to the sum of $d_1^{\ 2}$ and $d_2^{\ 2}$. This would be possible only if the interference terms in Eq.7 are of opposite sign to cancel each other, requiring either t or r to assume negative value or a π-phase jump in only one of the two directions. And classical electrodynamics tells us that it is the "external" reflection that undergoes the required π-phase jump [6]. When d_1 and d_2 are equal and t and r are $\sqrt{0.5}$ (a 50% beam splitter), one can engender 100% reflection or 100% transmission with the right choice of the delay number $\nu\tau$ along with the negative sign for the "external" reflection.

We are underscoring this "trivial" undergraduate physics to make the point that one cannot redirect a "photon" in an interferometer without the mediation of the beam splitter boundary molecules receiving light waves from both the directions with the "external" reflection condition experiencing a π-phase jump. Bell's theorem for "single photon interference" must incorporate this critical contribution of the classical dielectric boundary [Ch.6 of 2] for interference with Poynting vectors collinear. The simultaneous presence of real light waves from both the directions on the beam splitter boundary is essential to generate the energy re-direction capability of a passive dielectric boundary (for the "interference" effects to become manifest). That even (non-absorbing) passive material dipoles actually dictates re-direction of EM field energy is well known in classical physics from the explanation for the Brewster angle. At this angle ($\tan\theta_B = n$), the reflection of a beam becomes zero when the incident state of polarization is parallel to the plane of incidence because the E-vector induced undulation inside the medium (refracted direction) becomes parallel to the direction of reflection and dipoles cannot radiate along its axis of undulation [11]. Tracking the fringe position shift, due to this π-phase jump for non-collinear spatial fringe set up, is a tedious experimental task.

4. LOOKING DEEPER INTO LIGHT-MATTER INTERACTIONS BY VARYING DIFFERENT LIGHT PARAMETERS

4.1. Generic fringe visibility function for two-beam superposition

Let us develop our formulations for two well formed collimated light beams with steady frequencies, steady linear polarizations and steady phases. Their duration is longer than our recording time and hence the fields can be treated as continuous (CW). The cases for shorter duration of light pulses have been treated in an earlier paper [4]. Traditionally such a pair of beams will be called coherent. However, if the two beams have different frequencies or same frequencies with polarizations orthogonal, the fringe visibilities will be zero when we use slow detector. We should not assign "coherence" or "incoherence" properties to light beams without reference to detecting devices. When the parameters of a light beam (frequency, polarization, amplitude and phase) vary with time, the detector is forced to record a time average result based on it's over all intrinsic time constant of integration. If the rate of fluctuations of the composite field parameters is much slower than our detector's response time, we will be able to record time varying visibility of the fringes. For example, if we use a pico second streak camera and the field parameters are stationary for the duration of tens of pico seconds or longer; the camera will display fringes of time varying visibility. But, if under the same conditions of field fluctuations, we use a detector and a recorder with response times in the nano second domain, the recorded fringe visibility will be poor or zero. Obviously, the "coherence" of the optical fields has not become zero. Waves are a collective phenomenon; and they must have phase steady undulations at least over a couple of cycles to display their frequency parameter while interacting with quantum devices.

The fringe visibility for generic two beam superposition (different frequencies and polarizations), explicitly recognizing the linear first order susceptibility, can be derived as follows, where ${}^{(1)}\hat{\chi}_x$ are the unit dipole vector undulations induced by the electric vectors $\vec{a}_x$:

$$\begin{aligned} D(t) &= \left| {}^{(1)}\chi[{}^{(1)}\hat{\chi}_1 a_1 e^{i2\pi\nu_1 t} + {}^{(1)}\hat{\chi}_2 a_2 e^{i2\pi\nu_2(t+\tau)}] \right|^2 \\ &= {}^{(1)}\chi^2[a_1^2 + a_2^2 + 2a_1 a_2({}^{(1)}\hat{\chi}_1 \cdot {}^{(1)}\hat{\chi}_2)\cos 2\pi\{(\nu_1-\nu_2)t - \nu_2\tau\}] \\ &= {}^{(1)}\chi^2 A[1 + V\cos 2\pi\{(\nu_1-\nu_2)t - \nu_2\tau\}] \end{aligned} \tag{8}$$

The fringe visibility has to be measured out of the sinusoidally oscillating fringes in time.

$$V = 2a_1 a_2 \cos\varphi \big/ [a_1^2 + a_2^2]; \qquad {}^{(1)}\hat{\chi}_1 \cdot {}^{(1)}\hat{\chi}_2 = \cos\varphi \tag{9}$$

If the two beams of Eq.8 carry the same frequency and same state of polarization, then the detected fringes are time independent. Most of our interferometers work under this condition if the light is not pulsed.

$$D = {}^{(1)}\chi^2 A[1 + V\cos 2\pi\nu\tau] \tag{10}$$

Because visibility parameter in Eq.8 and 9 can be expressed without reference to the dipolar properties of the interacting materials, we have been safely ignoring the experimental fact that it is the joint stimulation of the material dipoles that really produce the fringes due to the superposed light beams. That is why this paper attempts to extract more information about the nature of light by explicitly focusing on the light-matter interaction processes.

4.2. Light-matter interactions for different polarizations.

If the two states of linear polarizations are parallel to each other, then $\cos\varphi = 1$ in Eq.9. If they are orthogonal to each other, the fringe visibility becomes zero as is well known; we have intensity without sinusoidal undulations. We should underscore that this continuous change in fringe visibility from unity to zero, based on the orientation angle between the two electric vectors, is mediated by the $\cos\varphi$ factor and is due to the unique behavior of the "sensing" material dipoles. In Section 3 we have underscored how a beam splitter can play active role in re-directing the energy from one beam into the other if the Poynting vectors are collinear. Of course, under this alignment conditions, the visibility has to be measured by scanning one of the mirrors to introduce relative phase delay τ . The same situation applies for polarized beams also, as long as the angle between the two polarizations is less than 90^0. When the Poynting vectors are non-collinear, τ varies with spatial locations and one can record fringes with a CCD camera or photographic plate.

But, why does fringe visibility ("coherence"?) decreases monotonically as $\cos\varphi$ and goes to zero for 90^0? Obviously the "coherence" properties (steady relative phase difference) of the two light beams have not changed. It is the dipolar properties of the materials that must dictate our observations. For all angles below 90^0 the dipoles simultaneously respond to all the E-vectors carrying complex amplitudes before absorbing energy according to $\cos\varphi$ law (Eq.9). Note that this is not Malus' $\cos^2\theta$ law of energy transmittance by a linear polarizer. When the two E-vectors are orthogonal, the dipoles cannot respond simultaneously to both the complex amplitudes; they separately respond to one or the other, generating zero-visibility intensity record.

4.3. Different possible models for E-vector-dipole response for superposition of two beams with same optical frequency.

While deriving Eq.9 we have assumed that the dipoles directly respond to both the E-vectors following the mathematical vectorial dot product rule. The superposed light beams remain independent of each other, as we are claiming in this paper (non-interference of light). Let us now consider two alternate models to evaluate their suitability in explaining superposition effects. (i) *Light beams interfere*. Superposed E-vectors create resultant E-vector before interacting with material dipoles. (ii) *Material dipoles are polarized by the strongest E-vector*. Then the projection of the other E-vectors is taken along this polarized direction for joint stimulations. We are assuming that the detecting molecules are embedded in an isotropic medium. The case for polarized crystalline detecting medium will be discussed in section 4.3.

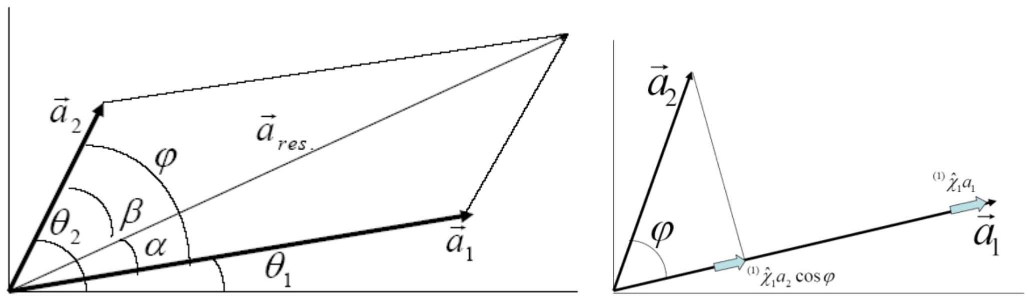

Figure 2. Case (i), left diagram: *Light beams interfere by themselves.* The two superposed beams with two different electric vector orientations are considered. For the case depicted on the left, a resultant E-vector is constructed by the two fields by themselves in the free space before they interact with any materials. **Case (ii), right diagram:** *Material dipoles are polarized by the strongest E-vector.* The polarized material dipoles then take the projections of the weaker E-vectors to create the resultant response.

4.2.1. Light beams interfere by themselves to create a resultant electric vector. The resultant field vector length and the complex amplitude can be expressed as (see Fig.2, left diagram):

$$\left|\vec{a}_{res}\right| = a_1 \cos\alpha + a_2 \cos\beta \quad \text{and} \quad \vec{a}_{res} = \hat{a}_{res} a_1 \cos\alpha e^{i2\pi\nu t} + \hat{a}_{res} a_2 \cos\beta e^{i2\pi\nu(t+\tau)} \tag{11}$$

Here $\hat{a}_{res}$ is a unit vector along the resultant vector $\vec{a}_{res}$. The angles α and β are made by the vectors $\vec{a}_1$ and $\vec{a}_2$ with the resultant vector $\vec{a}_{res}$. Let us assumed that the detecting molecules are isotropic and the resultant E-vector $\vec{a}_{res}$ dictates the direction of dipole stimulation. The light detecting molecules are now undulating along the direction $\vec{a}_{res}$ represented by the unit susceptibility vector ${}^{(1)}\hat{\chi}_r$. The detected intensity variation by changing the relative path (phase) delay τ between the two superposed beams can now be given by:

$$\begin{aligned} D &= \left|{}^{(1)}\chi\, {}^{(1)}\hat{\chi}_r (a_1 \cos\alpha e^{i2\pi\nu t} + a_2 \cos\beta e^{i2\pi\nu(t+\tau)})\right|^2 \\ &= {}^{(1)}\chi^2 ({}^{(1)}\hat{\chi}_r \cdot {}^{(1)}\hat{\chi}_r)[a_1^2 \cos^2\alpha + a_2^2 \cos^2\beta + 2a_1 a_2 \cos\alpha \cos\beta \cos 2\pi\nu\tau] \\ &= {}^{(1)}\chi^2 \{a_1^2 \cos^2\alpha + a_2^2 \cos^2\beta\}[1 + V \cos 2\pi\nu\tau] \end{aligned} \tag{12}$$

The visibility is now:

$$V = 2a_1 a_2 \cos\alpha \cos\beta / [a_1^2 \cos^2\alpha + a_2^2 \cos^2\beta] \tag{13}$$

Unlike for the case of Eq.8, the energy absorbed (or re-directed) is less than the total incident energy due to the multiplicative $\cos^2$ factors as if Malus' law is working. This is because of our choice of first creating a resultant E-vector using their geometric component from vector-sum algebra. The visibility of the fringes is reduced not only due to the unequal amplitudes $a_1^2 \cos^2\alpha$ and $a_2^2 \cos^2\beta$, but also by the cosines of the two angles the two original electric vectors make with the resultant E-vector direction $\vec{a}_{res}$.

We know from basic experiments that when the two E-vectors are orthogonal to each other ($\varphi = \alpha + \beta = 90^0$), even coherent beams cannot produce interference fringes. But the visibility here depends upon the product of the two cosines of α and β. Since neither α nor β can ever be 90^0, the visibility can never be zero. Hence this model of *"light beams interact (interfere) to form a resultant E-vector"* can be safely rejected.

4.2.2. Material dipoles are polarized by the strongest E-vector. Here also we start with the assumption that our detecting molecules are isotropic and can respond to all E-vectors oriented in any and all directions but are overridden by the strongest E-vector. This model implies that the detecting molecule gets polarized by the stronger E-vector $\vec{a}_1$ and the stimulated amplitude and direction is ${}^{(1)}\chi\, {}^{(1)}\hat{\chi}_1 a_1$ along the original vectorial direction of $\vec{a}_1$. The polarized and undulating molecule then takes a projection of the E-vector $\vec{a}_2$ along its existing undulating direction ${}^{(1)}\hat{\chi}_1$ with strength ${}^{(1)}\chi\,{}^{(1)}\hat{\chi}_1 a_2 \cos\varphi$ (see Fig.2, right diagram). So the intensity detected by the molecules will vary with the delay τ as:

$$\begin{aligned} D(t) &= \left| {}^{(1)}\chi\,{}^{(1)}\hat{\chi}_1 [a_1 e^{i2\pi\nu t} + a_2 \cos\varphi e^{i2\pi\nu(t+\tau)}] \right|^2 \\ &= {}^{(1)}\chi^2 (\hat{\chi}_1 \cdot \hat{\chi}_1)[a_1^2 + a_2^2 \cos^2\varphi + 2a_1 a_2 \cos\varphi \cos 2\pi\nu\tau] \\ &= {}^{(1)}\chi^2 \{a_1^2 + a_2^2 \cos^2\varphi\}[1 + V\cos 2\pi\nu\tau] \end{aligned} \tag{14}$$

Now the visibility is:

$$V = 2a_1 a_2 \cos\varphi \big/ [a_1^2 + a_2^2 \cos^2\varphi] \tag{15}$$

Eq.15 is plotted in the left diagram of Fig.3. Notice that the visibility relations given by Eq.9 and Eq.15 are very similar in the numerator, but their denominators are different. In this case, the qualitative observation that the fringe visibility reduces with the angle φ between the superposed E-vectors, is correct. However, the rate of reduction in the visibility is not monotonic except when the ratio of the two amplitudes corresponds to unity. This model of light matter interaction is also rejected as will be shown by careful experiments presented in Section 5.

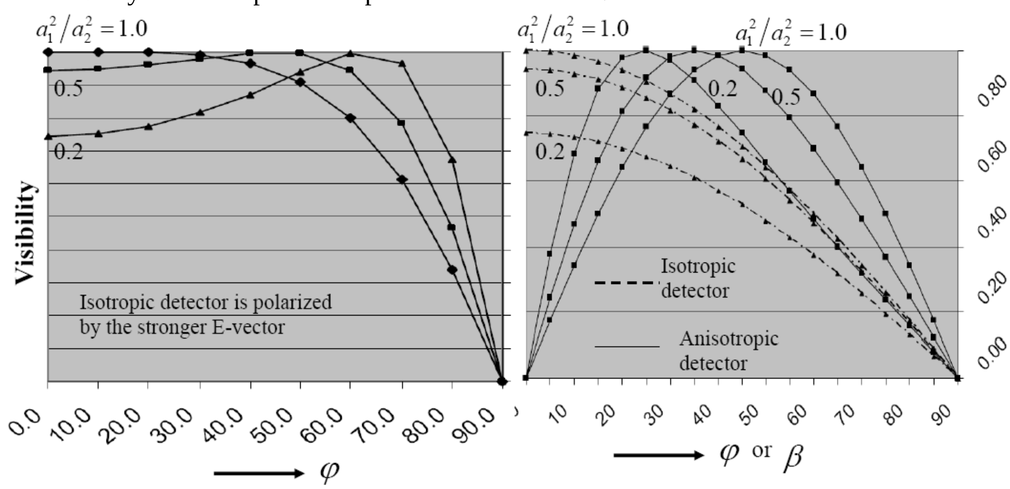

Figure 3. Theoretical visibility curves for different light-matter interaction models with the ratio of the intensities of the two superposed beams varying as 1.0, 0.5 and 0.2. **Left diagram:** The three curves correspond to the case where the detecting molecules are first polarized by the stronger of the two superposed E-vectors; the effective strength of the second E-vector is taken as a cosine projection. **Right diagram:** The set of curves with dashed-line correspond to the model that isotropic detecting molecules respond to all the superposed E-vectors simultaneously. The set of solid-line curves correspond to the model where the detecting molecules are embedded in a linearly polarized matrix.

4.2.3. Light beams remain independent. Isotropic detecting molecules respond to both the electric vectors simultaneously without preference. The relevant equations for this model have already been derived in the introduction of the Section 4, Eq.8, 9. If some detecting dipoles are in an isotropic medium and happen to be within the volume of physical superposition of the two beams, they will experience simultaneous stimulation by the two beams and the transformation it undergoes is effectively due to the summation of the two simultaneous joint stimulations. The angle between the two E-vectors is $\varphi \equiv (\theta_2 - \theta_1)$; where θ_2 and θ_1 are the polarization angles for the two E-vectors with reference to the horizontal direction. As before, we are assuming that the dipoles have first order linear susceptibility coefficient to polarizability due to any E-vectors field as ${}^{(1)}\chi$ .The visibility Eq.9 is reproduced here for reader's convenience:

$$V = 2a_1 a_2 \cos\varphi \big/ [a_1^2 + a_2^2]; \qquad {}^{(1)}\hat{\chi}_1 \cdot {}^{(1)}\hat{\chi}_2 = \cos\varphi \tag{9}$$

The theoretical plots for values of a_1^2 / a_2^2 corresponding to 1.0, 0.5 and 0.2 are shown in Fig.3 (right diagram) as three "dashed-line" curves. *We believe this is the correct model for light matter interaction.* Experimental results are presented in Section 5. When the electric vectors are exactly orthogonal, the dipoles separately respond to one or the other vector. No superposition effects can be registered by isotropic molecules even though light beams are coherent. We do not need an ad-hoc definition that orthogonally polarized light does not interfere.

4.3. Detecting molecules are embedded in an anisotropic medium with the polarized axis in a preferred direction.

Now, let us assume that we have an anisotropic detector where the detecting molecules are constrained to undulate only in the preferred direction $\vec{p}$ that makes angles α and β with the two E-vectors $\vec{a}_1$ and $\vec{a}_2$, respectively. Then the stimulating amplitudes that will be experienced by the anisotropic molecule, by *Malus' law for amplitude*, are $\vec{a}_1 \cos\alpha$ and $\vec{a}_2 \cos\beta$. Consideration of this model is relevant because of the advent of crystalline nano photonic and photo-EMF detectors.

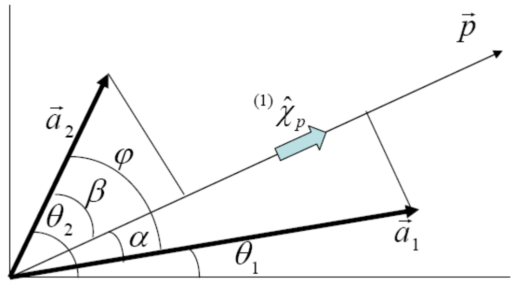

Figure 4. Detecting molecules are embedded in a polarized (or crystalline) medium such that their dipolar undulation is allowed only in its polarized direction.

Then the intensity registered by this anisotropic detector will be:

$$\begin{aligned} D_p &= \left| {}^{(1)}\chi\, {}^{(1)}\hat{\chi}_p (a_1 \cos\alpha e^{i2\pi\nu t} + a_2 \cos\beta e^{i2\pi\nu(t+\tau)}) \right|^2 \\ &= {}^{(1)}\chi^2 ({}^{(1)}\hat{\chi}_p \cdot {}^{(1)}\hat{\chi}_p)[a_1^2 \cos^2\alpha + a_2^2 \cos^2\beta + 2a_1a_2 \cos\alpha \cos\beta \cos 2\pi\nu\tau] \\ &= {}^{(1)}\chi^2 \{a_1^2 \cos^2\alpha + a_2^2 \cos^2\beta\}[1 + V \cos 2\pi\nu\tau] \end{aligned} \tag{16}$$

The visibility is now given by:

$$V \equiv 2a_1a_2 \cos\alpha \cos\beta \big/ [a_1^2 \cos^2\alpha + a_2^2 \cos^2\beta] \tag{17}$$

Notice that the fringe visibility is now reduced by two cosine factors, cosines of the angles made by the preferred polarized direction of the detecting material with the two E-vectors. The theoretical plots for values of a_1^2 / a_2^2 corresponding to 1.0, 0.5 and 0.2 are shown in Fig.3 (right diagram) as three "solid-line" curves. We have considered the case where the two incident E-vectors are orthogonal to each other implying $\alpha + \beta = 90^0$. If either α or β equals zero (the detector polarized axis lined up with one of the E-vectors), then the other angle must be 90^o and the visibility becomes zero, as is obvious from the solid- line curves. The maxima for the visibility now depends both on the ratio a_1^2 / a_2^2 and the specific values of α and β. Experimental validation is presented in Section 5.

4.4. Model to understand elliptically polarized light beam.

The key theme of the paper has been that well formed light beams do not interact with each other [1]. We also know that orthogonally polarized coherent light beams in two beam interferometry produce only zero-visibility fringes [3]. Then the physical reality of the mathematically congruent equation for elliptically polarized light needs to be re-visited. Can we really create light beams with elliptically spinning electric vector by superposing two coherent but orthogonally polarized light beams with unequal amplitudes having $\pm\pi/2$ relative phase difference in free space? It is a standard custom to represent an elliptically polarized light beam by the Jones vector comprising of two "coherent" orthogonal components with $\pm\pi/2$ phase shift [7]:

$$\left(E_{elip.}(t)\right) = \begin{pmatrix} \vec{a}_{1\cdot x} \\ \vec{a}_{1\cdot y} e^{-i\pi/2} \end{pmatrix} e^{i2\pi\nu t}; \;\; \text{Intensity} = a_{1x}^2 + a_{1y}^2 \tag{18}$$

The equation for the ellipse is derived by eliminating the common phase factor $\exp(i2\pi\nu t)$:

$$\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2\cos\varepsilon \frac{E_x E_y}{a_x a_y} = \sin^2\varepsilon; \;\; \vec{E}(z,t) = E_x \hat{x} + E_y \hat{y} \tag{19}$$

Note that "coherent" orthogonal amplitude components of the Jones' "vector" are never summed first to find the resultant intensity of the superposed fields. The intensity always remains the same by summing the squares of the two orthogonal component amplitudes. If, according to Eq.19, the "tip" of the E-vector were really helically tracing out an ellipse, then the instantaneous intensity should have been changing during every cycle of light propagation. Mathematical prescriptions of Jones' vectors and Jones' transfer matrices refer to light-matter interaction in the optical components and systematically propagate $\vec{a}_{1\cdot x}$ and $\vec{a}_{1\cdot y}$ independent of each other and the intensity is calculated from the final modified components by first squaring the x- and y-components and then summing them. Thus, this Jones' matrix approach preserves the observational science, while preserving the illusion that the E-vector of light beams by themselves can execute helical rotations! Since Muller matrix method stays focused always on the intensities, this methodology is also correct for scientific observations.

4.5. Light-matter interactions for different optical frequencies

We know that Michelson's Fourier transform spectrometry works under the assumption that light beams corresponding to different optical frequencies do not interfere with each other [8]. This assumption remains observationally correct as long as we use time integrating (slow) detectors [4,9]. However, after the invention of fast optical detectors [10], heterodyne beat signal detection by mixing different optical frequencies has become a very important applied tool for us. Again, the point is that light beams do not interact (interfere) with each other; the non-interference of light should not be reserved for specific parametric values of light beams. Let us superpose two optically steady beams with two different frequencies with two states of polarizations. For simplicity in mathematical representation, we are using dipole undulation vectors as $\vec{d}_x \equiv {}^{(1)}\chi^{(1)}\hat{\chi}_x a_x$.

4.5.1. Isotropic fast detector with broad transition bands for response to both frequencies. The electric fields for the two superposed beams make an angle φ between them. The case somewhat is similar to that of Section 4.2.3 and Eq.9, however, since the phase factor is proportional to the product $\nu\tau$ and we have two different frequencies, we need to keep track of delays experienced by each beam separately for accounting convenience and mathematical symmetry:

$$D(t) = \left|\vec{d}_1 e^{i2\pi\nu_1(t+\tau_1)} + \vec{d}_2 e^{i2\pi\nu_2(t+\tau_2)}\right| = (d_1^2 + d_2^2)[1 + V\cos 2\pi\{\delta\nu t + (\nu_1\tau_1 - \nu_2\tau_2)] \quad (22)$$

$$V \equiv 2d_1 d_2 \cos\varphi / [d_1^2 + d_2^2]; \qquad \delta\nu \equiv (\nu_1 - \nu_2) \quad (23)$$

This represents time varying cosine beat frequency fringes moving at a rate of $\delta\nu$ with visibility reduction factor V containing $\cos\varphi$ due to the angle between the two linearly polarized E-vectors. This expression is very similar to the case for same frequency beams (Eq.9), except that the fringes are now time varying and hence the visibility can be measured only electronically. The factor $(\nu_1\tau_1 - \nu_2\tau_2)$ is a fixed relative time delay between the two beams if the delays τ_1 and τ_2 are kept steady. Detecting time varying heterodyne fringes require special attention to keep the beam Poynting vectors as collinear as possible. A finite angle ϕ between the two beams will create laterally moving spatial fringes. Assuming $\bar{\nu}$ or $\bar{\lambda}$ as the mean frequency or wavelength of the two beams, spatial fringe spacing is given by $(\bar{\lambda}/2\sin\phi)$ or $((c/\bar{\nu})/2\sin\phi)$. Then the active size of the detector should be at least an order of magnitude smaller than this fringe spacing [9].

4.5.2. Anisotropic fast detector with broad transition bands for response to both frequencies. This case is similar to the case considered in Section 4.3 with the addition that the frequencies are also different. In absence of polarized detecting molecules, the experiment can be simulated by inserting a linear polarizer in front of an isotropic detector. Accordingly, the strengths of the dipole stimulations experienced by the detecting molecules $\vec{d}_1$ and $\vec{d}_2$ should be multiplied by the Malus' amplitude projection factors $\cos\alpha$ and $\cos\beta$. This projection makes $\vec{d}_1$ and $\vec{d}_2$ collinear along the polarizer direction and the vectorial direction is denoted by the common unit vector $\hat{d}$.

$$\begin{aligned} D(t) &= \left|\hat{d}(d_1\cos\alpha e^{i2\pi\nu_1(t+\tau_1)} + d_2\cos\beta e^{i2\pi\nu_2(t+\tau_2)})\right|^2 \\ &= (d_1^2\cos^2\alpha + d_2^2\cos^2\beta)[1 + V\cos 2\pi\{\delta\nu t + (\nu_1\tau_1 - \nu_2\tau_2)\}] \end{aligned} \quad (24)$$

Time varying visibility is:

$$V \equiv 2d_1 d_2 \cos\alpha\cos\beta / [d_1^2\cos^2\alpha + d_2^2\cos^2\beta] \quad (25)$$

Eq.25 is identical to Eq.17 for the case of same frequency. As before, the heterodyne fringe visibility can be measured only through record of time varying fringes.

5. EXPERIMENTAL RESULTS

5.1 Poynting vectors collinear (scanning fringe mode). Cases for isotropic and polarized detecting molecules.

When coherent light beams of same frequency but of different states (angles) of polarizations are superposed collinearly, both the beam-combining beam splitter BS2 [see MZ of Fig.1] and polarized detecting molecules can modify energy re-direction and transmittance, respectively, due to superposition of complex amplitudes due to the two beams. The representative visibility equations are given by Eq.9 and 17, respectively. But since the Poynting vectors are collinear, the fringes have to be recorded by scanning one of the MZ mirrors to introduce variations in the relative delay τ . The oscilloscope traces of such fringes are shown in Fig.5a and b. (3.1 and 4.1). In the absence of a crystalline polarized detector, we have simulated the case by inserting an analyzer in front of a usual isotropic detector. These qualitative results are presented simply to underscore the point that passive beam splitters and polarizers are capable of producing undulatory intensity changes due to the "complex amplitude stimulations" induced by the two light amplitudes of varying relative phase delay.

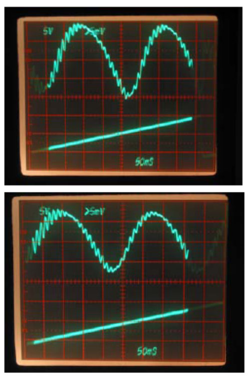

Figure 5. Scanning sinusoidal fringes due to energy re-direction by a beam splitter or modified transmittance by an analyzer with superposed collinear beams having two different polarizations. **Top picture:** Fringes produced due to a beam splitter. The angle between the polarization directions is 10^0; separate intensities are approximately equal. **Bottom picture:** Fringes produced due to a combination of the beam splitter and the analyzer. Intensity ratio for the two beams was about 3. The analyzer axis was making 30^0 and 40^0 with the two polarization directions of the two beams. Since the two beams were not orthogonally polarized, the fringe visibility is more complex for the case of (b) than for (a) because the complex amplitudes generated by the beam splitter were changing due to phase delay scanning before being received by the analyzer.

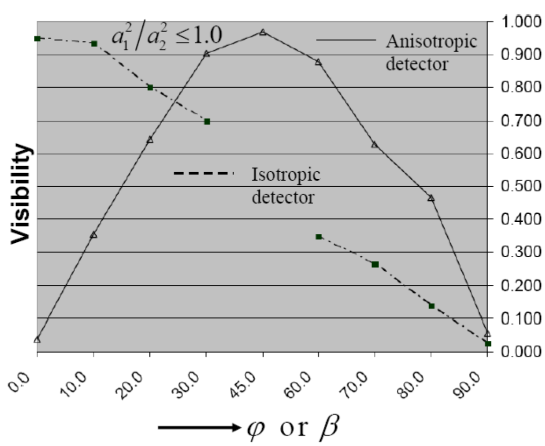

Figure 6. Measured visibility variation to be compared with the theoretically predicted dashed-line and solid-line curves shown in Fig.3 right hand diagram. The **dashed line** is the case for an isotropic detector. The corresponding x-axis angle represents φ of Eq.9. The **solid line** is the case for an anisotropic detector. The corresponding x-axis angle represents β $(\alpha = 90^o - \beta)$. The polarization angle between the detector polarization and the E-vector. The angle between the two E-vectors is 90 degrees.

5.2. Poynting vectors non-collinear (spatial fringe mode). Cases for isotropic and polarized detecting molecules.

We have used a CCD camera with software to quantitatively measure the intensity distributions of the registered fringes. In Fig.6 the experimental data points connected by the dashed line corresponds to an isotropic detector with the angle between the polarization vectors $\vec{a}_1$ and $\vec{a}_2$ changing from 0^o to 90^o with a_1^2 / a_2^2 slightly less than 1. This curve should be compared with the theoretically predicted one, the top dashed-line curve of Fig.3 drawn by using Eq.9. The match is within experimental errors.

The data points connected by the solid line corresponds to an anisotropic detector with the angle between the polarization vectors $\vec{a}_1$ and $\vec{a}_2$ kept fixed to 90^o, β was varied from 0^o to 90^o where $\alpha + \beta$ = 90^o and with a_1^2 / a_2^2 was slightly less than 1. This curve should be compared with the theoretically predicted one, the top solid-line curve of Fig.3

(right side) drawn by using Eq.17. The match is within experimental errors. In the absence of good anisotropic detector, we have simulated the condition by using an analyzer in front of the CCD camera (containing isotropic detector array).

These two experimental curves validate our core model of two beam superposition (interference) experiments with polarized light. While the mathematical expressions would not at all surprise the readers, the key point of this paper has been to underscore that the real intensity re-distribution becomes manifest when sensing material dipoles are able to interact simultaneously with the superposed light beams.

6. CONCLUSIONS

We have underscored the need to re-visit the phenomenon of superposition (interference) fringes in view of "non-interference of light" in general, and especially for arbitrary polarizations and carrier frequencies. Since superposition effects are displayed by the summing capability of sensing material dipoles induced in them by the multiple superposed light beams, we have developed the generic expressions for two-beam superposition effects in terms of fringe visibility (autocorrelation) functions. This approach of understanding "coherence" and "interference" phenomena in terms of material properties brings in conceptual simplicity and eliminates the need for non-causal explanations for various superposition effects. For example, re-direction of single indivisible photons, especially one at a time, in a collimated beam interferometer experiment, is conceptually difficult to appreciate because the asymmetric behavior of the isotropic beam splitter can be induced only in the presence of simultaneous presence of light beams on the beam splitter from both sides. Even though the mathematics "works" in most of the cases, the formulation of coherence theory in terms of Fourier frequencies has many embedded problems since the Fourier frequencies are not available for physical interactions [11,12,Ch.6 of 2]. We have shown that since coherence is measured in terms of fringe visibility V , it is better to derive it in terms of correlations of the dipole stimulations of the responding medium rather than field-field correlations. This will bring more "coherence" in understanding and distinguishing between *spectral coherence* and *temporal coherence* that give rise to the more complex *spatial coherence*. We are, of course, treating light beams as classical waves and sensing molecules as quantum devices, which has been popularized as semi-classical approach by Jaynes [13].

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