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Analysis of spectrometric data and detection processes corroborate photons as diffractively evolving wave packets

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ABSTRACT

In a previous paper [1], we have proposed that photons are diffractively evolving classical wave packet as a propagating undulation of the Complex Cosmic tension Field (C^2TF) after the excess energies are released by atomic or molecular dipoles as perturbations of the C^2TF. The carrier frequency of the pulse exactly matches the quantum condition $\Delta E_{mn}=h\nu_{mn}$ and the temporal envelope function creates the Lorentzian broadening of the measured spectral lines. In this paper we present both semiclassical and QM theories of emission and compare the QM prescribed natural linewidth of emitted spectra and the Doppler free laser absorption spectra to further validate our photon model.

Key words: Photons, Linewidth, emitted radiation, Lorentzian profile, spectrometer time- constant, divisible photon wave packet.

1. INTRODUCTION

Einstein first introduced the concept of light quanta in 1905 when he realized that the entropy of radiation in a cavity varies linearly with volume [3, 4]. The following quote is taken from his translated paper [3].

It seems to me that the observations associated with blackbody radiation, fluorescence, the production of cathode rays by ultraviolet light, and other related phenomena connected to with the emission or transformation of light are more readily understood if one assumes that the energy of light is discontinuously distributed in space. In accordance with the assumption to be considered here, the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.

The photon concept is more than a century old but both classical and quantum theories of radiation do not provide us with a prescription for modeling the shape and structure of an EM radiation emitted or absorbed by atoms and dipoles. Einstein himself seems to have had struggled with questions about light quanta [20] as indicated in his writing toward the end of his life: " All fifty years of conscious brooding have brought me no closer to the answer to the question: What are light quanta? Of course today every rascal thinks he knows the answer, but he is deluding himself."

The quantum theory of interaction between matter and field, Quantum Electrodynamics (QED), has been successfully applied to matter and field interactions. Despite the great success of QED in predicting matter field interactions outcomes, QED does not provide us with the nature and structure of a photon [12, 17]. QED does not also give us guidelines for localizing photons. Semi-classical radiation theory has also been successfully used in predicting experimental results but there are disagreements in some of the experimental results [19]. Our experimental data are guided and dictated by the interaction process between atoms, dipoles and fields. Atoms and dipoles as detectors guide our observation of emission and absorption processes. Developing a model of the emission process requires visualization of the invisible interaction process during emission.

In a previous paper [1], we have proposed that photons are diffractively evolving classical wave packet as a propagating undulation of the Complex Cosmic Tension Field (C^2TF) after the excess energies are released by atomic or molecular

dipoles as perturbations of the C^2TF. The carrier frequency of the pulse exactly matches the quantum condition $\Delta E_{mn} = h\nu_{mn}$ and the temporal envelope function creates the Lorentzian broadening of the measured spectral lines. In this paper we will compare and contrast the QM prescribed natural line width of emitted spectra and the Doppler free laser absorption spectra to further validate our photon model. In section 2 we discuss the spectrometric process to emphasize the relationship between observed fringes and the spectrometer time constant. In section 3, we present semiclassical theory of spontaneous emission and interpretation of linewidth. In section 4, we present quantum mechanical treatment of spontaneous emission and linewidth. In section 5, we present our photon model and discuss interpretation of linewidth. Some Doppler-Free linewidth measurements are also discussed. In section 6, we summarize our discussion of linewidth and photon wave packet.

2. SPECTROMETRY

The discovery of laser and the development of laser spectroscopy has provided us with increasingly precise measurements of atomic and molecular emission and absorption spectral lines [10, 11, 14]. In any standard classical spectroscopic treatment, a Fourier monochromatic wave referred to as continuous wave-CW is incident on spectrometer-Fabry-Perot or N slit Grating. A very basic and simplified version of the spectrometric process is shown in Figure 1. The intensity distribution on the screen is given by the Continuous Wave (CW) formula [6]

$$I_{cw}(\nu,\tau) = \frac{1}{N^2}\frac{Sin^2(N\pi\nu\tau)}{Sin^2(\pi\nu\tau)} \quad . \tag{1}$$

The CW formula for the intensity distribution of fringes assumes that the incident radiation is monochromatic- a wave that is infinite and continuously distributed over space. This violates conservation of energy and is inconsistent with the finite intensity fringes it produces. An alternative treatment of the spectrometric process given by Roychoudhuri, et al. [2] considers any classical spectroscopic instrument as having its own instrumental time constant $\tau_0 = N\tau$. In a N slit grating, for example, if a single short pulse light with frequency ν is incident on the slit, the grating replicates N finite pulses separated by time τ. For Fabry-Perot this time delay is given by 2d/c. For the N-slit diffraction grating it is dsinθ/c. The intensity distribution in Eq.(1) has been shown in reference [2] to be a special case of time-integrated intensity distribution of finite pulse in the limit as the pulse width approaches the time constant of the spectrometer $\tau_0 = N\tau$. The grating divides the radiation field into N fields with the same frequency. The fields each with its own delay introduced by the grating arrives at the detector-whose dipoles then sums the these fields to produce the intensity profiles that we observe.

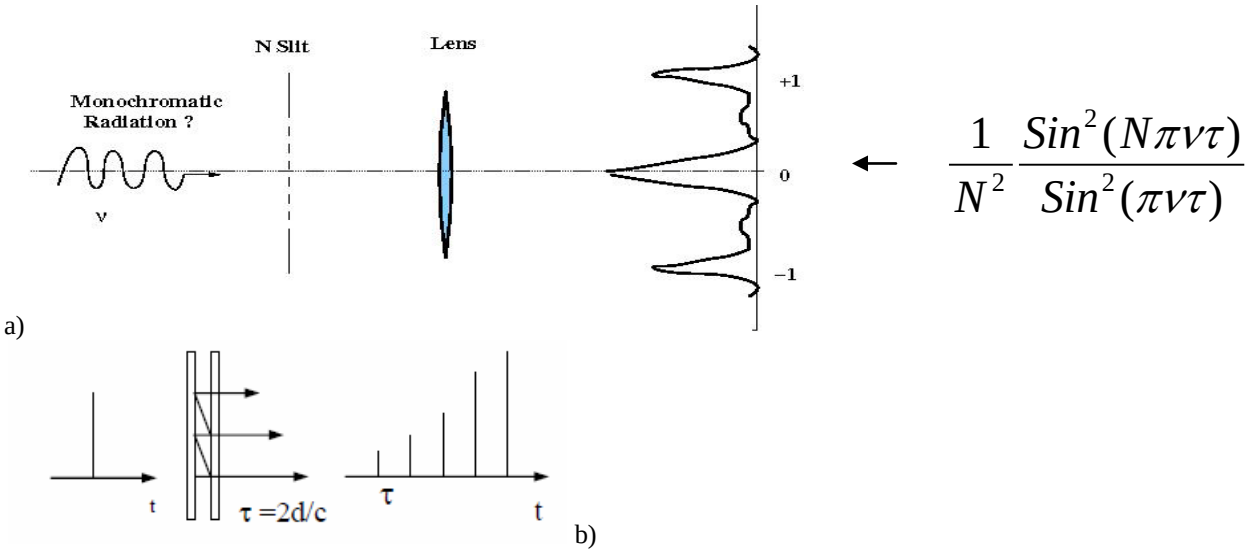


Figure 1. A schematic diagram for (a) N slit diffraction grating and (b) Fabry-Perot Interferometer.

The fringe intensity distribution as a function of time is given by the square modulus of the sum of the partially superposed train of pulses [2]

$$\left|i_{out}(t)\right|^2 = \left|\sum_{n=0}^{n=N} \frac{1}{N} d(t-n\tau) e^{i\pi\nu(t-n\tau)}\right|^2 \tag{2}$$

Where d is the dipole stimulation response to the radiation field. The time integrated fringe energy distribution is given by

$$I_{pls}(\nu,\tau) = \frac{1}{N} + \frac{2}{N^2}\sum_{p=1}^{N-1}(N-p)\gamma(p\tau)\cos[2\pi\nu\tau] \tag{3}$$

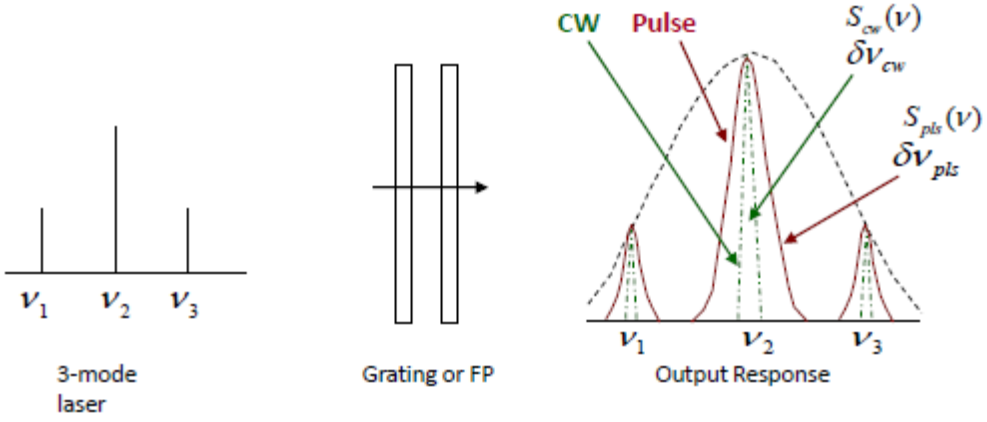
where p=|n-m| and

$$\gamma(p\tau) = \frac{\int d(t-n\tau)d(t-m\tau)dt}{\int a^2(t)dt} \tag{4}$$

is the normalized autocorrelation function. In the limit when the pulse width is larger than the spectrometer time constant τ_0, which is the size of the N-slit in the case of a diffraction grating, they recover the CW expression for the intensity distribution of the fringes[2] given by

$$\underset{PulesWidth\to N\tau}{I_{pls}(\nu,\tau)} = I_{CW}(\nu,\tau) = \frac{1}{N^2}\frac{Sin^2(N\pi\nu\tau)}{Sin^2(\pi\nu\tau)}. \tag{5}$$

Detailed work and expression for the Fabry-Perot interferometer can be found in reference [2]. The result in Eq. (5) implies that the width of the fringe intensity distribution is a function of the spectrometer time constant τ. If a short pulse with a pulse width less than Nτ is incident on the grating, then the fringe intensity distribution would be wider than the CW intensity distribution as shown in Figure 2. The fringe width decreases as the pulse width increases. The relationship in Eq.(5) also implies that the variation of the width in intensity distribution is not a consequence of new Fourier frequency of the intensity spectrum of the pulse. The pulse has a single frequency ν and the grating replicates the pulse but does not introduce any new frequency.



a) b) c)

Figure 2: a) Frequency spectrum of a 3-mode laser, b) Fabry-Perot Interferometer and c) Intensity distribution of the 3-mode laser.

The intensity distribution shown in Figure 2c is as a convolution of the Lorentzian profile and Gaussian profiles and is referred to as Voigt profile [10]. Deconvolution of the Voigt profile gives Lorentzian intensity profile. The value of the half- width at half-maximum (HWHM) of the Lorentzian profile is the natural linewidth of the spectral line. The natural linewidth for ^{87}Rb D-line transition ($5\ ^2S_{1/2} \rightarrow 5\ ^2P_{3/2}$) is about 6MHz [11, 13, 15]. The linewidth measurement from Doppler free saturation absorption dips reported in [13] for ^{87}Rb and ^{85}Rb are $\Delta\nu = 22 \pm 1\text{MHz}$ and

$\Delta\nu = 20 \pm 1\text{MHz}$, respectively. These results are in disagreement with the linewidth of 6MHz predicted by quantum mechanics.

3. LINEWIDTH: SEMICLASSICAL APPROACH

The discussion presented here closely follows that of reference [7]. If we assume an excited atom as a damped harmonic oscillator with a natural frequency ν_0 and damping constant γ, then we may write

$$\ddot{x} + \gamma \dot{x} + \nu_0^2 x = 0 \tag{6}$$

where x(t) is the position of the oscillator. The damping constant is given by $\gamma = \frac{2}{3}\frac{\nu_0^2}{c} r_0$ with $\gamma << \nu_0$ where r_0 is the radius of the electron. Imposing the initial condition $x(0) = x_0$ and $\dot{x}(0) = 0$ and assuming small damping constant for atomic oscillators, we may write the amplitude of oscillation to be

$$x(t) = x_0 e^{-\gamma t/2} e^{-i\nu_0 t}. \tag{7}$$

The frequency ν_0 corresponds to the central frequency of an atomic transition from an initial energy level E_i to a final energy level E_f and is given by $\nu_0 = (E_i - E_f)/h$. The amplitude of the oscillation decreases exponentially. For atomic transition this would mean that the emitted radiation is no more monochromatic. Instead, it is a superposition of all monochromatic frequency distributions. The amplitude of the emitted radiation also decreases with time and can be represented as a superposition of oscillators of different frequencies

$$E(t) = E_0 e^{-\gamma t/2} e^{-i\nu_0 t}, \tag{8}$$

Taking the Fourier transform of Eq. (8) gives us the amplitude of oscillation

$$E(\nu) = \int_{-\infty}^{\infty} E(t)\, e^{-i\nu_0 t}\, dt = \frac{1}{2\pi}\int_0^{\infty} E_0\ e^{-\left(\frac{\gamma}{2}\right)t} e^{-i(\nu_0 - \nu)t}\, dt \tag{9a}$$

which gives

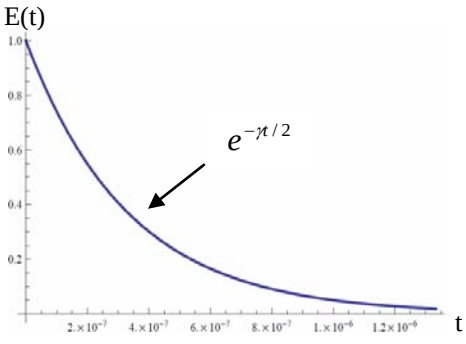
$$E(\nu) = \frac{1}{2\pi} E_0 \frac{1}{i(\nu - \nu_0) + \gamma/2}\ . \tag{9b}$$

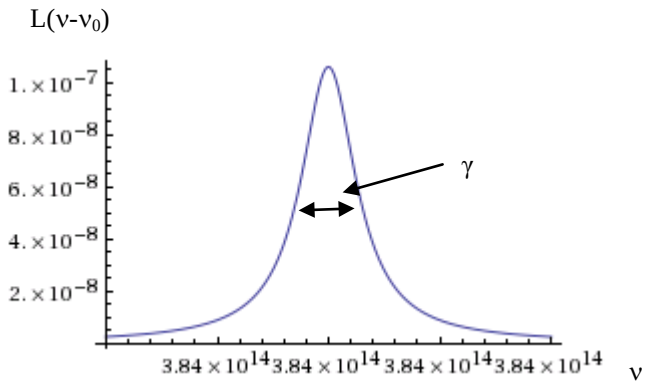
The intensity of the emitted radiation is proportional to $|E(\nu)|^2$ and can be expressed as

$$I(\nu) \cong \frac{I_0}{2\pi} \frac{\gamma}{(\nu - \nu_0)^2 + (\gamma/2)^2} \tag{10}$$

The normalized Lorentzian intensity profile is defined as $L(\nu-\nu_0) = I(\nu-\nu_0)/I_0$ [10]. The normalized Lorentzian is then

$$L(\nu - \nu_0) = \frac{1}{2\pi} \frac{\gamma}{(\nu - \nu_0)^2 + (\gamma/2)^2} \tag{11}$$





(a) (b)

Figure 3. (a) An exponentially decaying electric field amplitude and (b) Lorentzian intensity profile using Eq.(11). We have used the resonance frequency of Rubidium ^{87}Rb ($\nu_0 = 3.84\times10^{14}$ Hz and $\gamma = 6$ MHz). The Lorentzian spectral line profile shows that the full-half-width at half maximum (FWHM) is γ.

4. LINEWIDTH: QUANTUM MECHANICAL APPROACH

In Quantum mechanics atoms and electrons are represented by a wave function [8, 9]. Observable results are computed as mean values of the operators representing the observables. Linewidth of a spontaneous emission of radiation from an atom making one of its allowed transitions is proportional to the transition probability corresponding to this transition. Following the discussion in reference [7], we consider a two level atom with energy levels E_a and E_b to help us obtain a relationship between linewidth and transition probability. If we denote the transition amplitudes by b_{b0} and $b_{a1\lambda}$ with the initial condition that $b_{b0}(0) = 1$ and $b_{a1\lambda}(0) = 0$, then to first order approximation in the perturbation theory, we have

$$\begin{aligned} \hbar\dot{b}_{b0} &= \sum_{\lambda} H_{b0|a1\lambda} b_{a1\lambda} e^{i(E_\lambda - E_a - k_\lambda)t/\hbar} \\ i\hbar\dot{b}_{a1\lambda} &= H_{a1\lambda|b0} b_{b0} e^{i(E_a - E_b + k_\lambda)t/\hbar} \end{aligned} \tag{12}$$

Assuming solutions of the form $b_{b0}(t) = e^{-\gamma t/2}$ where γ is assumed to be the transition probability per unit time. This leads to a solution for $b_{a1\lambda}$ of the form

$$b_{a1\lambda}(t) = -H_{a1\lambda|b0}\frac{e^{i(\nu_\lambda - \nu_0)t - \gamma t/2} - 1}{\hbar(\nu_\lambda - \nu_0 + i\gamma/2)} \ . \tag{13}$$

The probability of finding the system in its ground state at time t is given $\left|b_{a1\lambda}(t)\right|^2$. For time $t >> E_b/\hbar$ (or time much greater than its lifetime 1/γ) the atom has jumped to the ground state by releasing radiation of energy $h\nu_\lambda$. Thus, at time $t \to \infty$, the probability that an energy of $h\nu_\lambda$ has been emitted is $\left|b_{a1\lambda}(\infty)\right|^2$. The intensity of the emitted radiation is proportional $\left|b_{a1\lambda}(\infty)\right|^2$ and is given by (detailed work can be found in reference [7])

$$I(\nu) = \frac{1}{2\pi}\frac{\gamma}{(\nu - \nu_0)^2 + (\gamma/2)^2} \ . \tag{14}$$

The intensity distribution expression in Eq.(14) obtained under quantum mechanical treatment is the same Lorentzian expression given in Eq.(11) that was obtained by considering a damped classical harmonic oscillator. In the classical treatment the linewidth γ is interpreted as a damping constant of the damped harmonic oscillator atom that leads to the exponential decay of the energy and amplitude of the emitted radiation. In quantum mechanics the linewidth γ is interpreted as the spontaneous transition probability per unit time. Using uncertainty principle $\Delta E \Delta t = \hbar$ it can be seen that γ is related to the width of the energy level E_b as $E_b = \hbar\gamma$. This relation implies that sharper energy levels have shorter lifetime. An extensive discussion of semiclassical and quantum theories of radiation can be found in [18, 21].

5. LINEWIDTH: PHOTON WAVE PACKET MODEL

Both standard classical and quantum treatments presented in the previous sections do not provide us a model for the emitted EM radiation. We now look at the model of emitted EM radiation wave packet proposed in [1]. The emitted EM radiation has a unique frequency ν and energy $\Delta E_{mn} = h\nu_{mn}$. It cannot continuously spread at all points in space because of conservation of energy [3]. Spontaneous emission of EM radiation is a time finite process. Thus, a photon model of the emitted radiation as an EM wave would be expected to have a finite temporal width of unique frequency ν.

In our proposed photon model, the emitted EM wave packet of frequency ν is modeled by a rapidly rising power and an exponential decaying function as shown in Figure 4. The electric field representing the wave packet envelope is given by

$$\vec{E}(t) = \vec{a}(t)\cos 2\pi\nu t = \mathrm{Re}\left[\vec{a}(t)e^{i2\pi\nu t}\right] \tag{15}$$

The rapidly rising and exponential dying amplitude $a_r(t)$ model is given by

$$a_r(t) = t^r e^{-t/2\tau} \tag{16}$$

where r =0.05 and $\tau = 1\mathrm{ns}$. More analysis and study is needed to establish the range and values of r. The pure exponential $a(t)$ used here is given by $a(t) = e^{-t/2\tau}$. The Lorentzian intensity profile of the emitted radiation follows from the time finite exponential decaying model of the emitted radiation. For EM radiation emitted with unique frequency, the Fourier transformation does not introduce a new EM frequency or a frequency spread [2].

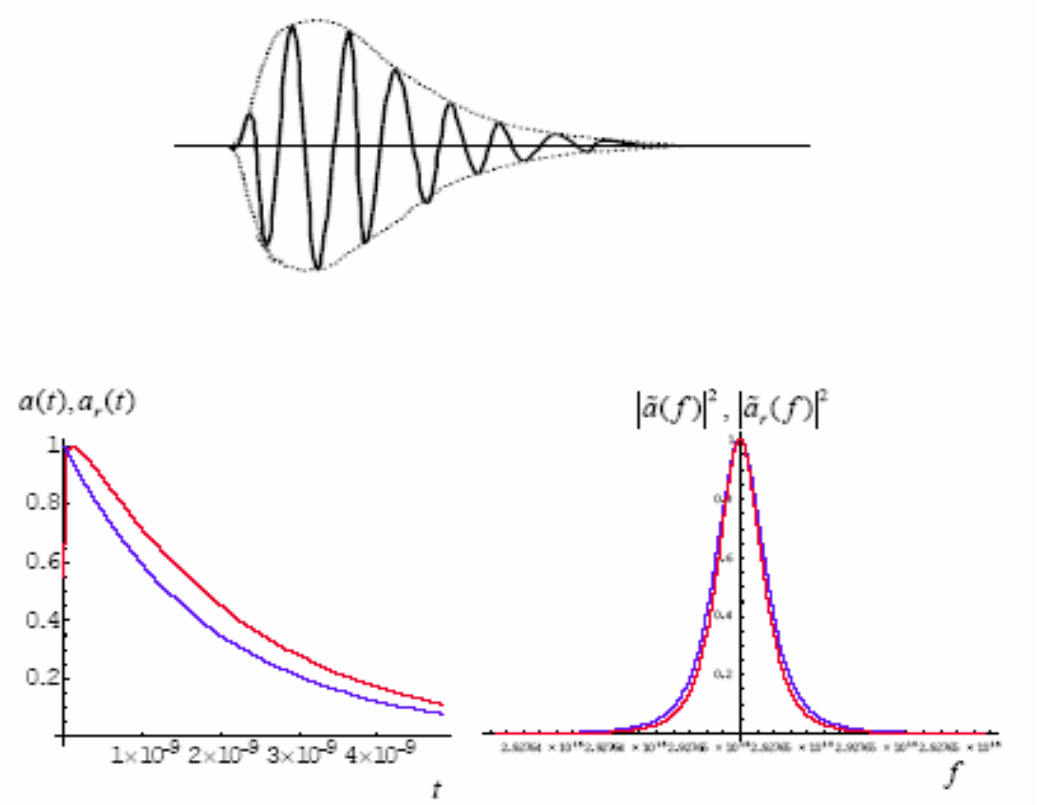


Figure 4. Top: A model for a rapidly rising and exponentially dying photon wave packet envelope with carrier frequency ν. Bottom Left: Represents photon wave packet amplitude; pure exponential (lower curve) and rapidly rising but exponentially dying (upper curve). Bottom Right: Fourier transform of the pure exponential has a slightly larger FWHM (upper curve) than the rapidly rising but exponentially dying amplitude model (lower curve). In computing, we have used the frequency of red cadmium light as the carrier frequency for the photon wave packet.

6. SUMMARY

Almost all spectroscopic measurements of emitted intensity distributions involve extracting the Lorentzian profile from a broadened intensity profile [15]. Some of the sources of the broadening are due to Doppler (thermal), collision, and optical power in laser spectroscopy. In any atomic emission, the emitted radiation is time finite with a unique carrier

frequency ν and finite energy packet hν imparted to it by the emitting atom. The physical model of the emitted energy $\Delta E_{mn} = h\nu_{mn}$ and its evolution into a classical wave packet is still missing. A distribution of EM radiation over all possible frequencies would require infinite energy and violates conservation of energy. The Lorentzian spectral broadening is commonly attributed to the frequency distribution as implied by Fourier Transform. In the photon model presented here the Lorentzian linewidth is attributed to the exponentially decaying photon wave packet envelope function that results in a Lorentzian profile but based on the shape of the emitted EM wave packet.

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