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Exploring light-matter interaction processes to appreciate various successes behind the Fourier theorem!

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ABSTRACT

This paper leverages many successes of the Fourier theorem in modeling the measurable experimental data in the fields of optical physics while underscoring some of the misinterpretations by focusing on the light-matter interaction processes, which give rise to the measurable data. We are proposing that we need to introduce Interaction Process Mapping Epistemology (IPM-E) to complement the currently successful Measurable Data Modeling Epistemology (MDM-E). We show that IPM-E helps us recognize that EM waves cannot produce interference fringes by themselves as they do not interact with each other. Accordingly we have been missing the NIW-principle (Non-Interaction of Waves). Application of the NIW-principle to Fourier theorem, as applied to optical physics reveals and helps us understand many optical phenomena much better than so far we have understood. We discuss Fourier optics, Fourier transform spectrometry, coherence theory, spectrometry theory and laser mode locking theory and summarize for each case, the deeper understanding that we have been missing by neglecting the NIW-principle.
Keywords: J. W. Goodman, NIW-principle, Fourier optics, Fourier transform spectrometry, Coherence theory, Spectrometry of short pulses, Mode lock theory.

1. INTRODUCTION

First, I must thank the organizers for arranging these multi disciplinary talks as a tribute to Prof. Goodman who facilitated the entry of a great number of young researchers to feel comfortable to venture into a newly emerging field of Fourier Optics in USA through the publication of his mathematically accurate and yet a lucid book in 1966 for any self starter [1]. Second, I am grateful that I have been given the opportunity to present my views regarding the applications of the Fourier theorem in diverse phenomena of optical physics in front of such a distinguished group of scientists. My views are deliberately constructed as provocations towards challenging the inquiring minds and the innovators to think out-of-box and accelerate the deeper understanding of the various light-matter interaction processes and hence enhance the rate of growth in optical physics and technologies. I hope my modest provocative attempt will add some value to the field of optics, which is now openly recognized as the broadest enabling discipline, providing with empowering tools, both scientific and technological, to advance all other branches of science and engineering.

Let me first summarize the key points, which will then be elaborated in the later sections:

⊙ **Fourier theorem (FT) is not a general principle of nature**: FT is probably the most successful and pervasively used mathematical theorem in all branches of physics and engineering. Its success in modeling diverse measurable data makes us believe the theorem to be a *defacto* principle of nature as evidenced by the pervasive use of the Fourier frequencies as if they physically exist in our linear measurement instruments and systems [2,3]. FT as a tool was originally invented about 200 years ago by Fourier to model the evolution of heat energy conduction process in a medium. Eventually, it found its way into almost every branch of physics.

⊙ **Measurable of Data Modeling Epistemology (MDM-E):** Our ongoing methodology of theorizing nature's working principles based upon our propensity of staying satisfied with modeling the experimental data, without delving into the underlying invisible light-matter and matter-matter interaction processes, can be characterized as MDM-E [4]. There are clear justifications for being complacent with MDM-E. We have not discovered any better model than MDM-E to validate nature's behavior with a theory. However, while emulating a success model brings more successes quite rapidly, staying in the rut of the same success model deprives us from allowing the evolution of our enquiring minds; and hence, learning deeper realities of nature! We have been suppressing our innate capability of visualizing the invisible interaction processes in nature, which are giving rise to the measurable data that we are modeling so successfully with our theories!

Such restrictions in our thinking have been strongly encouraged by the Copenhagen Interpretation of Quantum Mechanics.

⊙ **Interaction Process Mapping Epistemology (IPM-E):** The key purpose of physics is to understand and visualize the various interaction processes in nature that give rise to the measurable data. Such visualization facilitates emulation of natural process to invent and to innovate newer and better technologies, which are at the root of accelerating the human evolution. This process also keeps our theories anchored to nature's objective reality, even as our knowledge of nature remains clearly incomplete [5]. We have been diverging away from how to visualize the invisible light-matter interaction processes because of our sustained successes through the application of MDM-E. While MDM-E will and must remain as the key approach to doing experimental science and engineering; we must now incorporate IPM-E as a tool for our next higher level of contemplation to map nature, even though our map may remain far from the exact reality for a very long time as we keep on evolving.

⊙ **"Measurement Problem" correctly identified by Quantum Mechanics:** It persists because we have been ignoring the interaction processes that give rise to the data. We can appreciate this point simply by logically dissecting the measurement process. We can measure only some physical transformations experienced by the interactants in our measuring device or system. Transformations must necessarily be preceded by some energy exchange. Energy exchange must be guided by some allowed force of interaction between the interactants. All forces being physical range dependent, interactants must be within each other's sphere of physical influence. Thus, all measurable interactions must necessarily be "local" [6, 7]. Neutrons and protons must be pushed to within femto meters to form nucleus; whereas the Sun and the Pluto can be held together even at distances of many many mega meters. Now, the question is whether we can gather all the necessary and complete information about the interactants by carrying out a large variety of measurements with the same pair of interactant. Our view is that the "measurement problem" is inherent in all of our measurements and hence this problem cannot be resolved by any elegant mathematical theorem. The point is further illustrated below.

⊙ **Incomplete Information Challenge (IIC):** Our sensors (detectors) can neither gather all the information about the interactants; nor can they transfer whatever limited information they gather. This is because we have not yet figured out how to make the interactants display all the necessary transformations in our instrument due to all the existing four natural four forces that are always active with some non-zero strengths. Further, our recorders (receivers) can never register 100% of whatever limited information is buried there in the registered transformations with complete fidelity [5-7]. We have been aware of this perpetual IIC, but neither do we explicitly acknowledge it, nor do we make our students aware of it. Genius scientists like Galileo, Newton, Einstein, Schrodinger, etc., have demonstrated how to fill up this information gap by constructing some brilliant hypotheses and present to us with working theories. But, these are theories in progress as they are necessarily incomplete, being constructed based upon insufficient knowledge of the universe. We still do not really know what a photon or an electron is!

⊙ **Overcoming IIC:** We are suggesting, with some examples in this article, that explicit recognition of IIC and application of IPM-E, while "riding on the shoulders" of successful MDM-E, we can iteratively refine, or modify, or correct, the foundational hypotheses behind all working & future theories. And then create more and more refined map of the cosmic logics and a working image of the "Cosmic Elephant". We need to humbly recognize that we are all "blind people" and our theories are built upon our highly individualistic interpretations of observed data (measured transformations), molded by our personal and subjective epistemologies [1, 4-7]. This recognition will help us appreciate Newton. We must humbly try to perpetually keep on increasing the horizon of human scientific knowledge by "riding on the shoulders of the giants", instead of assuming them as messiahs of final truth!

⊙ **Human logics, Mathematical Logics & Cosmic Logics:** Humans have achieved a staggering degree of successes in modeling the living biosphere and the non-living cosmo-sphere by, first, using *human logics* to create proper hypotheses and bring conceptual continuity among diverse observations by imposing some logical congruence. Then we apply *mathematical logics* to give our hypotheses the structure of a working theory, which succeeds in predicting new behaviors and measurable data in nature, and hence captures some of the operational *cosmic logics* behind interaction processes driving the cosmic evolution. It is useful to breakdown our logical processes behind theorization so we can keep on correcting at all levels the logical structures behind all working theories, which are always work in progress [4].

⊙ **Various Fourier theorems - SS-FT, TF-FT & TD-FT, and their applications:** We will define and then use three distinctly different forms of FT and then demonstrate the proposed IPM-E to accelerate the progress in physics [2,3]. Our methodology of thinking based upon our propensity of staying satisfied with modeling the experimental data using the FT without delving into the underlying invisible light-matter interaction processes has already been characterized as MDM-E above. For the convenience of later discussions, let us classify the various modes of using FT in different branches of optical physics: (i) SS-FT denotes space-space Fourier transform as in spatial Fourier transform of aperture functions used in optical imaging, signal processing, van Cittert-Zernike theorem. (ii) TF-FT denotes time-frequency Fourier theorem that takes the transform of the envelope function of a light pulse and propagates the Fourier frequencies through linear and non-linear optical devices and systems. (iii) TD-FT denotes time-delay Fourier transform, which takes the transform of the oscillatory component of an interferogram as in Fourier transform spectrometry; or taking the Fourier transform of the autocorrelation data due to a short laser pulse and obtain the Fourier intensity spectrum of the pulse (Wiener-Khintchine theorem). The acronyms, SS-, TF- and TD-, represent different pairs of Fourier conjugate variables, which correspond to different sets of physical parameters and should be identified with truly physical variables that map physical interaction processes, as underscored in later sections.

2. THE NIW-PRINCIPLE! IPM-E, IMPOSED OVER MDM-E, PROVIDES FURTHER CHARACTERITIC OF LIGHT, NEGLECTED OVER CENTURIES!

The logical flow diagram of Fig.1 easily reveals that when we impose IPM-E on our detection process, we can appreciate: (i) The Incomplete Information Challenge already defined in the last section, which has been imposed on us by nature. (ii) The generic behavior of all propagating waves that they can co-propagate or cross through each other while remaining unperturbed in their wave front energy distributions, which we are proposing as the NIW-principle (Non-Interaction of Waves) in the linear domain [8]. The extension of the NIW-principle on to FT clearly implies that this theorem requires special cares while applying it to model different physical phenomena depending upon the underlying physical processes.

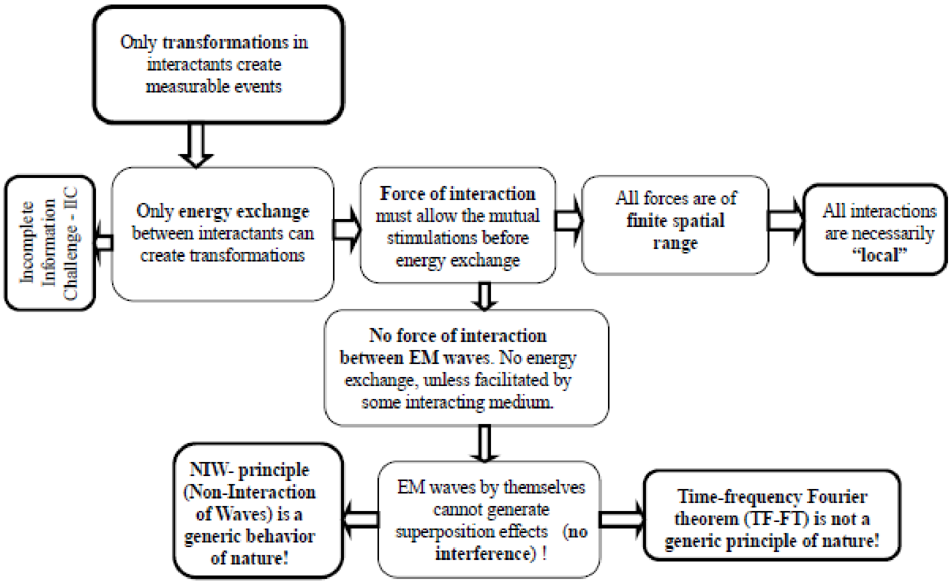

Figure1. The logic flow diagram demonstrates that imposition of IPM-E (Interaction Process Mapping Epistemology) onto the physical process that generates measurable data, lead us to recognize the NIW-principle (Non-Interaction of Waves). The NIW-principle also implies that the Fourier theorem of linear summation of harmonic waves cannot be a generic principle of nature.

2.1. Experimental demonstration of the NIW-principle

Multiple parallel beams, produced by a tilted but parallel pair of beam splitters (known as a Fabry-Perot spectrometer), made to converge on a one-sided ground glass with a focusing lens as shown in Fig.2 [9-11]. The original laser beam contains two frequencies (longitudinal modes). The flat front surface reflects out the convergent beams as a divergent set as if the reflection process from a flat surface does not allow the beams to experience each other's presence (non-interaction of waves)! The clusters of silica molecules on the back side of the ground glass become detectors & facilitate spectral energy separation leveraging superposition of multiple beams. Wherever the sum total linear stimulation of the silica clusters due to N-separate focused and superposed beams is zero for a particular optical frequency, the forward scattered light energy is negligible and we register a dark fringe. Clusters experiencing the resultant maximum stimulation provide the maximum forward scattering of light and we register a bright fringe for a specific optical frequency, when we image the ground glass surface. Thus, the actual carrier frequency of the incident light determines the bright spectral fringe. Note that the detector (here, silica clusters) must be within the physical volume of the superposed beams to help generate the fringes; underscoring that light matter interaction process to generate "interference fringes' must be a "local" phenomenon. The enlarged image of the ground glass surface gives spectrally resolved fringes for the laser modes. Note that if we were to send a pulse out of this laser beam through such a pair of parallel beam splitters, it cannot carry out the Fourier transform of the pulse envelope; it would simply replicate the incident pulse into a set of N pulses with a periodic delay between them. Light beams, by themselves, cannot re-distribute their energy!

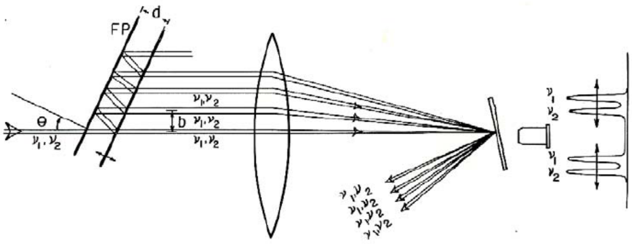

Figure 2. An elementary experiment demonstrates the NIW-hypothesis. The convergent beams are reflected from the back surface of a ground glass plate and diverge out unperturbed and unchanged, preserving all the original properties. But the scattered light from the forward surface display spectral separation and fringes due to superposition as energy re-distribution facilitated by the "lumpy" silica molecule assembly (from [11]).

3. APPLICATION OF THE NIW-PRINCIPLE TO SS-FT, TD-FT AND TF-FT

3.1. SS-FT or Space-Space Fourier Transforms

3.1.1. Optical Signal Processing: Space-space Fourier transform, which is at the root of optical image (signal) processing is a special case of the Huygens-Fresnel diffraction integral (Eq.1). The integral simply mathematically morphs into a spatial Fourier transform-like integral as the quadratic phase curvature, built into HF secondary wavelets, evolves into plane waves in the far-field, as shown in Eq.2, where the problem has been reduced to one dimensional version for conceptual simplicity. The Fourier kernel consists of a pair of space variables, ξ & x and hence named as space-space Fourier transform [1]:

$$U(P_0) = \frac{-i}{\lambda} \iint_{\Sigma} U(P_1) \frac{\exp(ikr_{01})}{r_{01}} \cos\theta \, ds \tag{1}$$

$$U(x) = \frac{e^{ikz} e^{i\frac{kx^2}{2z}}}{i\lambda z} \int_{-\infty}^{+\infty} U(\xi)\, e^{-i\frac{2\pi\xi x}{\lambda z}} d\xi = C \int_{-\infty}^{+\infty} U(\xi)\, e^{-i2\pi\xi f_x} d\xi; \;\; f_x = (x/\lambda z) \tag{2}$$

Since the HF-integral maps the physical process of "secondary wavelet" propagation. Hence, Fourier optics, or SS-FT conforms to a physical principle in nature. SS-FT is immensely successful in optical signal processing. Eq.1 & 2 map the reality of diffractive nature of light very closely. We should recognize that the use of the HF integral, or the direct use of the Maxwell's wave equation, provides us with the most remarkable tools of predicting cosmological phenomenon, like spatial coherence of star light, as well as nano phonics and plasmonic photonics dealing with light-matter interaction processes in the nano domain. Nobody in these fields propagates "indivisible photons"!

Let us now recognize that the linearity of the Huygens-Fresnel (HF) diffraction integral, representing linear sum of secondary HF wavelets, automatically incorporates the NIW-hypothesis [12]. We can compute the complex amplitudes at any forward plane at any distance z due to a diffracting aperture at z=0 only because each HF wavelet is allowed to evolve unperturbed by each other's presence even though they are expanding and evolving through the same space while partially overlapping with each other. These evolving wave amplitudes do not possess any inherent properties or capacities to re-organize or interact with each other to create resultant energy distribution (diffraction fringes). In the optical domain, the energy distribution can be observed only when we insert some material dipoles (detectors) within the volume of superposition and then they absorb energy proportional to the square modulus of the sum of all the superposed wave amplitudes, which is the QM provided recipe. So, if a CCD camera or a photographic plate is exposed to the complex amplitudes of Eq.2 for an interval of time ($t_2 - t_1$), then the registered energy distribution $E(x)$ can be represented as the integration over the exposure interval of the square modulus of Eq.2, multiplied by the polarizability of the detector χ :

$$E(x) = C^* C \int_{t_1}^{t_2} \left| \int_{-\infty}^{+\infty} \chi(\nu) \cdot U(\xi)\, e^{-i2\pi\xi f_x} d\xi \right|^2 dt = C^* C \chi^2 \int_{t_1}^{t_2} \left| \int_{-\infty}^{+\infty} U(\xi)\, e^{-i2\pi\xi f_x} d\xi \right|^2 dt \qquad (3)$$

If $\chi(\nu)$ remains constant for a narrow band of optical frequencies being used, then, according to our mathematical rules, it can be taken out of the square modulus and the two integration operations. Then the final step in Eq.3 appears to be taking directly the square modulus of the sum of all the HF wavelets at the desired plane. This has been giving us the illusion of "interference of light" even though the physical process of creating the "dark-bright" fringes is carried out by the detecting dipoles. Note that all the superposed HF wavelets contribute to the resultant energy transfer from the fields to the detectors. If we only pay careful and respectful attention to our working mathematical theories, we would not need to accept logically incongruent assumptions that the fringes are created due to arrival and non-arrival of indivisible single photons at the bright and the dark fringe locations, respectively! The energy transfer is due to the superposition of N-waves on to a detecting dipole. The energy transfer process physically materializes after the dipole executes the resultant dipolar undulation induced by N-electric vectors (detector sums the joint stimulation). The dipole experiences N-different amplitudes and N-different phases, or 2N different physical parameters and executes a single resultant oscillation before absorbing proportionate amount of energy from all the stimulating waves. Under no existing laws of physics a single stable and indivisible elementary particle can pick up 2N different physical information from diverse physical locations and makes itself appear and disappear at different forward locations. We do not find any logically over-riding reason to declare that the wave theory of superposition effect in the optical domain is invalid just because the detecting dipoles are quantized. The diffractive beam divergence being inversely proportional to the frequency of the waves, X-rays and Gamma-rays tend to appear as localized "bullets" delivering energy.

3.1.2. Spatial Coherence or van Cittert-Zernike Theorem: For the sake of completeness, we would like to mention that the far-field degree of spatial coherence due to an incoherent thermal source, like the Sun or a discharge lamp, is the Fourier transform of the source intensity distribution function. This is known as the van Cittert-Zernike theorem [13]. Again, the mathematical structure of the far-field coherence takes the semblance of a Fourier transform integral because the starting premise is the propagation of HF secondary wavelets from each point source independent of each other, which become plane waves, resembling Fourier transform kernel. The mathematical starting point is a physical principle.

Another point to note is that the enhancement of the fringe visibility in the far-field is not due to increase in the phase correlation between the independent point sources. As per the NIW-principle, they propagate without influencing each other. But, due to diffractive spreading of the individual point sources, each self-coherent wave front begins to cover wider areas. When one carries out a double-slit fringe-visibility experiment in the far field [13], each point source generates its own perfect visibility fringes, but spatially translated from each other because the source points are displaced from each other. Such displacement of two-beam cosine fringes $\cos 2\pi(\nu\tau)$ can be conveniently defined as translation of the fringe-order number, traditionally defined as $m = \nu\tau$; for vC-Z theorem, the delay τ is the variable which is determined by the different source positions [14]. The sum of these translated but perfect-visibility fringes produces a resultant degradation in the fringe visibility. The reader will find below that degradation of the fringe visibility for a source containing multiple frequencies, under time-integrated recording, is also dictated by a similar fringe translation relation $m = \nu\tau$, the variable now being ν .

3.2. TD-FT, or Time-Delay Fourier Transform

In this section we apply the NIW-principle to the concepts behind Fourier transform spectroscopy, the heterodyne spectroscopy, and the spectral and temporal correlation (or fringe visibility degradation) in two beam interferometry.

Unlike for SS-FT cases, the mathematical structure of the TD-FT, or time-delay Fourier transform, cannot be traced back to any fundamental physical principle. TD-FT spectroscopy works because the theory matches the data under time integrated detection after some mathematical manipulation. Thus, a working mathematical theory should be able to guide us to visualize the invisible light-matter interaction processes! Light beating spectroscopy reveals the physical significance of temporal integration of light induced measurable transformations in the detector. To significantly reduce the mathematical complexity and still preserve the key steps behind light-matter interaction process, let us assume that we are using a Michelson two-beam interferometer to carry out Fourier transform spectroscopy using a two-mode CW He-Ne laser [15]. The detector receives four beams, two each for the two modes. Then the detector current is given by:

$$D(t,\tau) = \left|\chi e^{i2\pi\nu_1 t} + \chi e^{i2\pi\nu_2 t} + \chi e^{i2\pi\nu_1(t+\tau)} + \chi e^{i2\pi\nu_2(t+\tau)}\right|^2$$
$$= \chi^2 \begin{bmatrix} 4 + 2\cos 2\pi(\nu_1 - \nu_2)t + \cos 2\pi\{(\nu_1 - \nu_2)t - \nu_2\tau\} \\ +2[\cos 2\pi(\nu_1 - \nu_2)(t+\tau) + \cos 2\pi\{(\nu_1 - \nu_2)t + \nu_1\tau\} \\ +2[\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau] \end{bmatrix} \quad (4)$$

A DC-coupled fast detector will display an oscillatory DC current riding on a DC-bias (first term), τ -dependent slowly varying current (two terms of the 3rd line) and rapidly varying heterodyne current proportional to the difference frequency $(\nu_1 - \nu_2)$. However, if we significantly enhance the LCR-integration time of the detector circuit, all the time dependent cosine terms will be reduced to zero and we will be left with terms what Michelson originally would have registered with his time integrating photographic plate:

$$\bar{D}(\tau) = \chi^2[4 + 2\{\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau\}] \quad (5)$$

CW laser modes are not "incoherent to each other"; otherwise the heterodyne terms would have been absent always. One should note that the degradation of the fringe visibility as denoted by Eq.5 happens because of translations of the fringes of order $m = \nu\tau$ corresponding to different frequencies for the same delay τ; not because the two CW laser modes are mutually phase incoherent to each other. Michelson's recipe is then to remove the "DC" signal and recover the normalized fringe visibility, also known as the autocorrelation function $\gamma_\nu(\tau)$ and then Fourier transformed this signal that oscillates with mirror translation delay τ. This mathematical step, as in Eq.6, then recovers the physical spectrum of the source $\tilde{D}(\nu)$ consisting of two longitudinal modes (frequencies):

$$\begin{aligned} D_{Osc.}(\tau) &= \cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau \\ \tilde{D}(\nu) &= \delta(\nu - \nu_1) + \delta(\nu - \nu_2) \end{aligned} \quad (6)$$

It is important to recognize that in the process of simplifying and manipulating the signal we have lost trace of χ, and the crucial physical role played by the detectors in creating the superposition effect! Further, Michelson actually jumped from Eq.4 to Eq.5 directly because his photographic plates were automatically carrying out the time-integration. High speed photo detectors were invented only during late 1950's. Michelson presented the physical hypothesis that different optical frequencies do not interfere with each other. We now know that the real physical process behind his assumption was the temporal integration of the detected signal by his detectors. The role of detectors in generating the observed superposition fringes is clearly corroborated by the NIW-hypothesis, presented by us.

Let us now note that $\gamma_\nu(\tau)$ and $\tilde{D}(\nu)$ form a Fourier transform pair, which is usually presented as Wiener-Khintchine theorem in most text books [13]. The generalized integral expression can be written as:

$$\gamma_\nu(\tau) = \int_0^\infty \tilde{D}(\nu)\, e^{+i2\pi\nu\tau} d\tau, \text{ where: } \tilde{D}(\nu) = \int_0^\infty \gamma_\nu(\tau) e^{-i2\pi\nu\tau} d\tau \quad (7)$$

We have deliberately written $\gamma_\nu(\tau)$ with a subscript ν to underscore that the fringe visibility degradation is due to the real presence of multiple physical frequencies in the light beam being analyzed.

We prefer to call $\gamma_\nu(\tau)$ as the spectral correlation function to distinguish it from the temporal correlation function $\gamma_t(\tau)$ which we use to explain the degradation in fringe visibility due to delayed superposition of a pair of replicated pulses through a Michelson-like two-beam interferometer. Here the degradation of fringe visibility arises due to superposition of unequal amplitudes rather than the presence of many physical frequencies. This can be appreciated from Fig.3a and 3b. A single pulse $a(t)$ is replicated by a Michelson-like two-beam interferometer and superposed with a delay τ. As the delay increases starting from zero, the instantaneous visibility of the fringes, as would have been registered by a super fast streak camera, continue to decrease and eventually becomes zero when the pulses are no longer overlapping at all. At varying separations, the superposed amplitudes due to the two pulses that stimulate the detector are varying, forcing the detector to register time varying fringe visibilities. When one integrates this time varying correlation due to the two pulses, the result is the decaying temporal autocorrelation, whose normalized expression $\gamma_t(\tau)$ is given by Eq.8. Note that the replicated pulse pair is definitely not "incoherent" to each other, just like two laser modes are not incoherent, as discussed earlier! Degradation of fringe visibility is determined by the detector's integration time. If we can invent an atto second detector, all light sources will register instantaneous high visibility fringes!

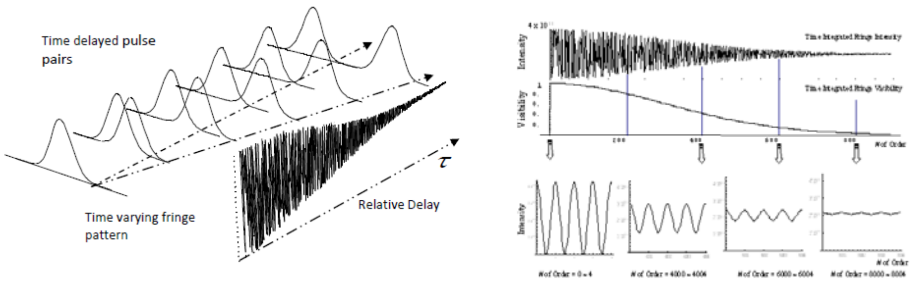

Figure 3. Visualizing the delay dependent partial superposition of a pair of replicated pulses (left) and the corresponding reduction of fringe visibilities (right) due to superposition of different amplitudes at any particular moment for different delays.

$$\begin{aligned}\gamma_t(\tau) &= \int \chi a^*(t)\chi a(t-\tau)dt \Big/ \left[\int |\chi a(t)|^2\, dt\right]^{1/2} \left[\int |\chi a(t)|^2\, dt\right]^{1/2} \\ &= \int a^*(t)a(t-\tau)dt \Big/ \left[\int |a(t)|^2\, dt\right]^{1/2} \left[\int |a(t)|^2\, dt\right]^{1/2}\end{aligned} \tag{8}$$

Note further that the final line of this normalized expression in Eq.8 is devoid of the detector's polarizability, giving us the impression that electromagnetic waves interact by themselves to generate the measurable correlation factor. By cancelling χ^2 from the numerator and the denominator, we have been deceiving ourselves from appreciating the key role played by the detectors in generating all superposition effects.

If $a(t)$ and $\tilde{a}(f)$ form a time-frequency Fourier transform pair:

$$a(t) = \int_0^\infty \tilde{a}(f)\, e^{-i2\pi ft} df \quad \text{and} \quad \tilde{a}(f) = \int_0^\infty a(t)\, e^{i2\pi ft} dt\,, \tag{9}$$

Then the Wiener-Khintchine theorem implies that the normalized temporal correlation function $\gamma_t(\tau)$ and the normalized Fourier spectral intensity function form another Fourier transform pair [14]:

$$\gamma_t(\tau) = \int_0^\infty |\tilde{a}(f)|^2_{norm}\, e^{-i2\pi\nu\tau} df \quad \text{and} \quad |\tilde{a}(f)|^2_{norm} = \int_0^\infty \gamma_t(\tau) e^{i2\pi\nu\tau} d\tau \tag{10}$$

We have purposefully added the subscript "t" to the temporal correlation function $\gamma_t(\tau)$ to underscore that the degradation in the fringe visibility is due to superposition of unequal amplitudes of two translated (delayed) pulses. Note also that we have used f to denote the mathematical Fourier frequencies derived from the pulse envelope $a(t)$ given by Eq.9. We would now request the readers to appreciate the differences between the two correlation functions $\gamma_\nu(\tau)$ as defined in Eq.7 based upon physical spectrum $\tilde{D}(\nu)$ and $\gamma_t(\tau)$ as defined in Eq.10 based upon mathematical spectrum $|\tilde{a}(f)|^2$. Note that the Fourier conjugate variables in Eq.9 are (t, f), but those for Eq.10 are (τ, f). There are neither any logical, nor any interaction process based (physical) connection between the parameters t and τ .

Physically relevant definition for $\gamma_t(\tau)$ is given by Eq.8; the definition in Eq.10, albeit being mathematically correct, confuses us of the physical origin of the degradation in fringe visibility. Note further that the derivation steps behind Eq.10 requires one either (i) to drop the cross terms between different frequencies, which implies "non-interference of waves of different frequencies", or (ii) to take the time integration over the entire duration of the pulse pair involved in the analysis, which implies the use of a detector having the built-in characteristic of taking a long-time integration of the signal. We have underscored these points earlier.

We should now recognize that TF-FT generated mathematical frequencies of a pulse envelope do not exist in reality! Further, such Fourier monochromatic frequencies supposed to mathematically exist for all time and hence violate the principle of conservation of energy. We must remain vigilant whenever we use Fourier mathematical frequencies to formulate the physical behavior of either linear or non-linear systems [2,3,16]. The next section illustrates this point by deriving a generalized theory of spectrometry by propagating a pulse with its carrier frequency, rather than the Fourier frequencies of the pulse envelope. We then show that the classical text book formula for CW light is a special case of our general derivation.

3.3. TF-FT, or Time-Frequency Fourier Transform in Spectrometry

All real signals generated in nature have to be space and time finite to conform to the law of conservation of energy. Even a CW laser must be turned on and off! The most general spectrometric response function must be derived due to a finite pulse, not due to a Fourier monochromatic mode just because the modeling is easy and because it seems to be working so far. Fig.4 helps visualize that our spectrometers like gratings and Fabry-Perto's replicate an incident pulse into a series of N-pulses, where N is the number of slits for a grating and the finesses number for a Fabry-Perot (FP) [17-22]. When the pulse is narrower than the step delay τ , the pulses do not overlap at all and cannot facilitate superposition effects on material boundaries or detectors. When the width is comparable to τ , instrumental sensing materials experience partial superposition of the train of pulses and generate fringes whose widths are broader than those that are produced when the duration of the pulse is greater than $\tau_0 = N\tau$, where we define τ_0 as the characteristic time constant of a spectrometer. Under this condition of a pulse broader than τ_0 our formula becomes identical to the classical CW formulation, still producing a finite width of the "spectral" fringes. We have been correctly calling this "CW fringe width" as instrumental response function and have been de-convolving this response function to obtain true spectral distribution function. Our theory gives similar, but a generic pulse response function that can be de-convolved to find the true spectral distribution function or the actual carrier frequencies contained in the pulse. We have found that our time integrated pulse response function can be expressed, using the Parseval's energy conservation theorem, as the convolution of the classical CW spectrometer response function with the mathematical Fourier spectral intensity function due to the pulse envelope function. We believe that because of this mathematical coincidence we have developed the tendency to assume that Fourier frequencies are real since it conforms to measurable data (recall MDM-E!). We can now appreciate that $\delta\nu\delta t \geq 1$ cannot be a principle of nature. We can simply de-convolve the pulse response function to achieve super-resolution [17,18].

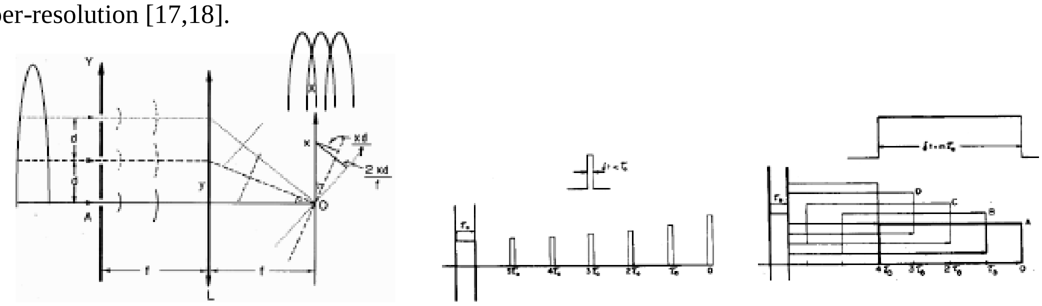

Figure 4. A grating or a Fabry-Perot spectrometer replicates an incident pulse into a train of N-pulses with a periodic delay set by the instrument. Superposition of the output pulse train depends upon the pulse width and their spacing.

The mathematical approach in formulating the pulse response function can be appreciated from the Fig.5. Appropriate material medium simply responds to the partially overlapped N-pulses. The consequent time varying irradiance is given by the Eq.11:

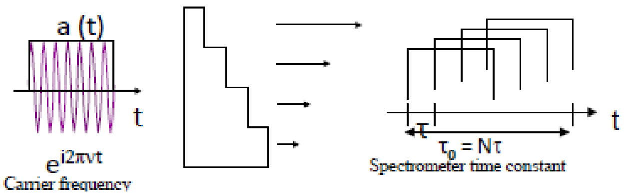

Figure 5. A detector array at the output slit of a spectrometer responds to N-periodically displaced pulses to generate frequency dependent energy distribution (spectral fringes) whose width is dictated by the artifact of partial superposition of pulses.

$$|\,i_{out}(t)\,|^2 = \left|\sum_{n=0}^{N-1}(1/N)\chi(\nu)a(t-n\tau)\cdot\exp[i2\pi\nu(t-n\tau)]\right|^2 \tag{11}$$

The traditionally recorded time integrated energy, or the pulse response function, is given by:

$$E_{pls}(\nu,\tau) = \frac{\chi^2}{N} + \frac{2\chi^2}{N^2}\sum_{p=1}^{N-1}(N-p)\gamma(p\tau)\cos[2\pi p\nu\tau] \tag{12}$$

Where the autocorrelation function between all possible pulse pairs is given by:

$$\gamma_t(p\tau) = \int a(t-n\tau)a(t-m\tau)\ dt \Big/ \int a^2(t)\ dt \tag{13}$$

Eq.14 presents the limiting value of Eq.12 when the width δt of $a(t)$ exceeds the characteristic time constant of the spectrometer $\tau_0 = N\tau$. It becomes identical to the classical CW formula for a grating as $\gamma_t(p\tau) = 1$. Of course, the term χ^2 does not exist in classical expression because of the wrongly held assumption that light beams interfere.

$$\underset{\substack{\delta t \to \tau_0 = N\tau \\ \gamma(p\tau)\to 1}}{Lt.} E_{pls}(\nu,\tau) = \frac{\chi^2}{N} + \frac{2\chi^2}{N^2}\sum_{p=1}^{N-1}(N-p)\cos[2\pi p\nu\tau] \equiv \frac{\chi^2}{N^2}\frac{\sin^2 \pi N\nu\tau}{\sin^2 \pi\nu\tau} \equiv E_{cw}(\nu,\tau) \tag{14}$$

It is a mathematical coincidence that the measured time integrated fringe broadening implies the existence of Fourier frequency of the pulse envelope when one uses Parseval's energy conservation theorem! $\tilde{A}(\nu)$ does not exist physically!

$$I_{pls}(\nu,\tau) \approx \int_{-\infty}^{\infty} \left|i_{out}(t)\right|^2 dt = I_{cw}(\nu) \otimes \tilde{A}(\nu);\ \text{where}\ \tilde{A}(\nu) \equiv \left|\tilde{a}(\nu)\right|^2 \tag{15}$$

3.4. TF-FT, or Time-Frequency Fourier Transform in Laser Mode Locking

TF-FT plays a critical role in mode-phase-locked pulse generation, but the physical pulses are generated by the phase-locker's time-gating property as in Eq.17, not by self-interference of the modes, as in Eq.16 [23]!

$$a_{mod.lck.}(\nu_0,t) = \sum\nolimits_{-(N-1)/2}^{+(N-1)/2} a e^{i2\pi(\nu_0+n\delta\nu)t+i\varphi_0} = a\frac{\sin N\pi\delta\nu t}{\sin \pi t\delta\nu} e^{i2\pi\nu_0 t+i\varphi_0} \tag{16}$$

To present the physical concept most simply, we have assumed that all the mode amplitudes are equal. We have also used the traditional cavity mode spacing relation $\delta\nu = c/2L$. If Fourier summation of phase-locked mode theory were correct, then the resultant EM wave inside the cavity would have been oscillating at the mean central frequency of all the modes. However, the fact that commercial femto second lasers deliver the famous "frequency comb" [24], the entire set of cavity modes must be co-propagating and remaining independent of each other (the NIW-principle) while acquiring intra-cavity gains. Eq.14 does not represent the physical reality of any laser behavior. It is the square-law response of the intra-cavity mode-locking material medium that provide the complementary time-gating function allowing a steady train of cavity energy to come out:

$$D_{tm.gate}(t) = \left|\sum\nolimits_{-(N-1)/2}^{+(N-1)/2} \chi a e^{i2\pi(\nu_0+n\delta\nu)t+i\varphi_0}\right|^2 = a^2\chi^2\frac{\sin^2 N\pi\delta\nu t}{\sin^2 \pi t\delta\nu} \tag{17}$$

We believe that if the NIW-principle were widely known, scientists, attempting to obtain the shortest possible pulses, would have discovered fs lasers, probably, a couple of decades earlier by staying focused on discovering the fastest time-gating light-matter interacting material, rather than discovering only the most spectrally broad lasing medium. We do need spectrally very broad gain medium most importantly because the lasing atoms in such a medium must recycle themselves at the fastest possible speed to provide rapid temporal gains competing with the rapidly repeating cavity energy losses.

4. SUMMARY

We have explained that the mathematical form of pace-space Fourier transform, as applied in optical signal processing and measurement of spatial coherence, arises out of the physical principle of propagating Huygens-Fresnel secondary wavelets. Time-delay and time-frequency Fourier transform relations do not have any foundational principle of physics. So, when they model measurable data, we must use their successes to visualize the invisible light-matter interaction processes. This approach also gives a better and deeper understanding of the nature of light and their interaction processes with our sensors (detectors). We have proposed the NIW-principle that light beams do not interact (interfere) with each other. We have validated our proposition by presenting better explanations of "coherence theory" as correlation of detector stimulations, because non-interacting waves cannot generate correlation signals by themselves. Since light beams do not interfere with each other, they should not be characterized as "coherent or incoherent". Detectors' energy absorption and integration times dictate the resultant fringe visibility. If we can invent an atto-second detector, it will always give "instantaneous" high visibility fringes from almost any light!

Our observations imply that the prevailing mode of doing science can be characterized as Measurable Data Modeling Epistemology (MDM-E) and we must continue to validate all our theories using MDM-E. However, we believe that we must now introduce a complementary epistemology, Interaction Process Mapping Epistemology (IPM-E), to look deeper into the invisible interaction processes that give rise to the measurable data. We believe our sustained evolution depends upon accelerated technology innovations by emulating nature's interaction processes, for which IPM-E will be an empowering mode of thinking.

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