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Discerning comb and Fourier mean frequency from a fs laser, based on the principle of non-interaction of waves

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ABSTRACT

The key objective of this article is to underscore that as engineers, we need to pay close attention in repeatedly validating and re-validating the underlying physical processes behind a working theory that models a phenomenon we are using to create tools and technologies. We use the test case, the prevailing mode-lock theory, to illustrate our views by identifying existing contradictions and showing approach towards their resolution by identifying the relevant physical processes. The current theory tells us that the Fourier summation of all the allowed cavity modes directly produces the train of pulses. It effectively assumes that electromagnetic (EM) waves are capable of re-organizing their spatial and temporal energy distribution to generate a train of temporal pulses while preserving the spatial mode energy distribution. The implication is that EM waves interact with each other by themselves. Even though the theory is *working*, we have three logical problems. First, in the real world, in the linear domain, waves never interact with each other. On careful analysis of all types of *interference* experiments, we will recognize that only in the presence of some interacting material medium can we observe the physical superposition *EFFECT*. In other words, detectors carryout the superposition effect we call *interference phenomenon*, through the summation of their multiple simultaneous linear stimulations and then absorbing energy proportional to the square modulus of the sum total stimulation. Second, a Fourier monochromatic wave, existing in all space and time, is a non-causal hypothesis. Just because our theories are *working* does not mean that we have understood the real physical interaction processes in nature. We need to build our theories based upon space and time finite EM wave packet containing a finite amount of energy, which is a causal approach. Third, in spite of staggering successes of Quantum Mechanics, we do not yet have a self consistent model for space and time finite model of a photon. QM only predicts that EM energy emission (spontaneous and stimulated) takes place only in a discrete amount at a time from atoms and molecules. It does not give us recipe about how to visualize a propagating photon as it expands diffractively. However, Huygens-Fresnel's classical diffraction integral gives us a rigorous model, which is the cornerstone of modeling evolution of laser cavity modes, CW or pulsed. In this paper, we highlight the contradictions that arise out of the prevailing mode-lock theory and resolve them by using causal models, already underscored above. For example, there are now a wide range of very successful technological applications of the frequency comb extracted out of fs lasers. If the Fourier summation were the correct physical process, then all the cavity modes would have been summed (converted) into a single mean frequency around the gain line center for perfectly mode-locked systems. Further, sending such fs pulses through an optical spectrometer would have always displayed a *transform limited* fringe, centering on the mean Fourier frequency, rather than generating the comb frequencies, albeit instrumentally broadened. Output pulse train from a phase locked laser is functionally produced due to the oscillatory time-gating behavior of the intra-cavity phase-locking devices. So, we need to pay more attention to the fast temporal behavior of the materials we use for achieving very fast time-gating, since this material imposes phase locking on the cavity modes to enhance its own high-contrast time-gating behavior.

Keywords: Mode-lock, Non-Interaction of Waves, Frequency comb, Transform limited spectrum, Fourier monochromatic waves, Model of photon, Temporal response of phase-locking material.

1. INTRODUCTION

A theoretical physicist will normally be content if a theory, proposed to explain a natural phenomenon, consistently matches relevant measurable data. Let us name this approach of thinking as the Measurable Data Modeling Epistemology (MDM-E). An engineer, on the other hand, needs to be consistently vigilant in constructing a causally self consistent map of the interaction processes behind the phenomenon s/he uses. We invent tools and technologies that must work in the macro causal world by effectively emulating the natural processes in novel ways to suit our requirements to survive and evolve with comfort. Let us call this model of thinking as the Interaction Process Mapping Epistemology (IPM-E)[1]. The prevalence of MDM-E is obvious; our measuring instruments provide us with reproducible data consistently. The difficulty with IPM-E is that no interaction process, whether we call it classical (hitting a base ball with a bat) or quantum mechanical (hitting a Ne-atom with an accelerated electron inside He-Ne laser discharge tube), is directly visible to our eyes or instruments in every microscopic details. Thus, IPM-E requires very creative visualization using our faculty of imagination. The key objective of this paper is to use the example of the prevailing mode-lock theory, which is *working* very well, and yet there exists conceptual contradictions. We will underscore several existing contradictions between experimental observations and the core concept behind the prevailing mode-lock theory. Then, we will recognize that if we force ourselves to visualize the physical processes behind mode-locking, i.e. the generation of a train of temporal pulses through phase-locking between the cavity longitudinal modes, our first attention should be on finding an intra-cavity device whose light-matter interaction process can provide the fastest possible oscillatory time-gating behavior, and the immediate second attention should be on the broadest possible gain-line band-width such that the lasing atoms (molecules) can be recycled through excitation, de-excitation and re-excitation at a speed faster than the time-gating oscillator.

A careful attention on light-matter interaction processes will reveal that waves do not physically interact (interfere) with each other in the linear domain. Otherwise, dozens of independently modulated laser beams multiplexed through the same single mode fiber could not have successfully reproduced all the independent channel-data on de-multiplexing the beam at the receiving station. The phenomenon of superposition is not just an abstract mathematical principle. We need to view it as the ***superposition effect***. *The superposition effect becomes observable in our instrument as some physical transformation (release of photo electrons, etc.) when the detector is capable of simultaneously responding to all the stimulating waves simultaneously.* The summation implied by the superposition effect is carried out by the detecting molecule as joint amplitude stimulation. Then the molecule carries out the energy absorption to display the measurable transformation, which follows the recipe of square modulus of the sum of all the dipole stimulations, correctly given by Quantum Mechanics.

This Non-Interaction of Waves (NIW)[2] questions the generic validity of summing electromagnetic waves to obtain new resultant energy distribution without the need of mediation by some detecting material (physical interaction). We know that to detect the heterodyne difference-frequency due to the superposition of two EM waves of different frequencies, we need a broad-band solid state detector (along with fast electronics); gaseous atoms with sharp energy levels cannot display the difference frequency. The quantum properties of the detectors determine the Superposition Effect that we can observe. The reader may now recognize that a simple Fourier summation of the allowed cavity longitudinal mode does not represent the physical process by which we obtain mode-lock pulse train. It is the broad-band response capability of the intra-cavity phase locker that becomes the oscillatory time-gate, generating intra-cavity reverberating pulses. The out-coupling mirror simply transmits a portion of this reverberating energy. The implication of the Maxwell's wave equation accepting any linear summation of sinusoidal waves as another acceptable solution is that, within the linear domain, multiple EM waves can co-propagate or cross propagate without perturbing (changing) each others' energy distribution. The same logic applies to the time-frequency Fourier theorem (TF-FT). Only interacting material media can display energy re-distribution due to superposed waves, provided its QM property allows it to respond to all the waves simultaneously.

The next issue pertains to "The nature of light: What are photons?"[3] Neither classical physics, nor quantum physics has provided us with any causal and self consistent model for photons as discrete packets of EM energy emitted (spontaneous and stimulated) by atoms and molecules, which propagate, spreading diffractively, accurately following, from macro to nano domains, the Huygens-Fresnel's classical model of diffraction integral. All cavity mode modeling and computations are based on some version of the classical HF integral; or the Maxwell's classical wave equation in complex material environment. We should recognize that EM waves in the optical domain always consist of time and space finite pulses. All of our basic theories of optical phenomenon should be based upon propagating finite pulses, not based upon propagating Fourier monochromatic modes spanning in all space and time, which does not exist in the real world. Even QED defines photons as Fourier modes of the *vacuum*.

With this background, we now identify the contents of various sections of the paper. Section-2 identifies the contradictions in observed data from mode-locked lasers. Fourier summation of cavity modes would imply that only the central mode frequency remains active inside the cavity, effectively suppressing all other cavity modes, which contradicts the existence of frequency comb (all the cavity modes). The reality of frequency comb is obvious from the wide variety of novel fundamental science and technological applications[4]. Section-3 proposes a model that can help visualize the realistic process of evolution of laser modes inside a cavity by propagating space and time finite wave packets emitted through spontaneous and stimulated emission by lasing atoms or molecules. Section-4 presents the summary of the theory of a spectrometer, based on propagating a photon wave packet, which should be used to analyze laser pulses. Section-5 presents a brief discussion of the issues presented in this paper.

2. CONTRADICTIONS IN THE OBSERVED DATA OF MODE-LOCKED LASERS

2.1 Traditional mode-lock theory

Consider a gain-flattened laser medium under consideration, as in Eq.1 (Fig.1a).Then the standard approach of Fourier summation[4,5] of the periodic mode amplitudes will be given by Eq.2, depicted in Fig.1b. The frequency comb of N-modes has been reduced to the single central frequency, which contradicts all observations see Fig.2 and 3.

$$S(\nu) = \sum_{-(N-1)/2}^{+(N-1)/2} \delta(\nu_0 + n\delta\nu) \tag{1}$$

$$E_{cavity}(\nu_0, t) = \sum_{-(N-1)/2}^{+(N-1)/2} e^{i2\pi(\nu_0 + n\delta\nu)t + i\phi_c} = e^{i2\pi\nu_0 t + i\phi_c} \sum_{-(N-1)/2}^{+(N-1)/2} e^{i2\pi(n\delta\nu)t} = \frac{\sin N\pi(t/\tau)}{\sin \pi(t/\tau)} e^{i2\pi\nu_0 t + i\phi_c} \equiv a(t - n\tau) e^{i2\pi\nu_0 t + i\phi_c} \tag{2}$$

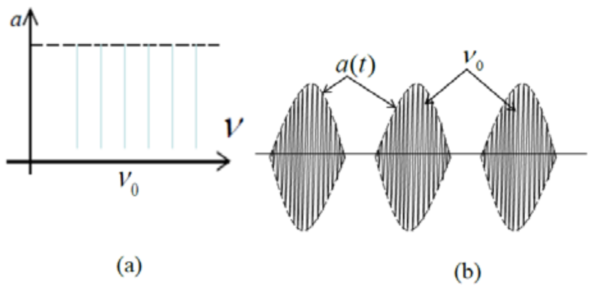


Figure 1. Traditional mode-lock theory obtains the pulse train by taking the square modulus of the sum of the complex amplitudes of all the allowed modes. Implication is that the EM waves interact by themselves to re-organize the field energy as discrete pulses while the cavity mode frequencies are replaced by a single carrier frequency, which is the allowed central mode.

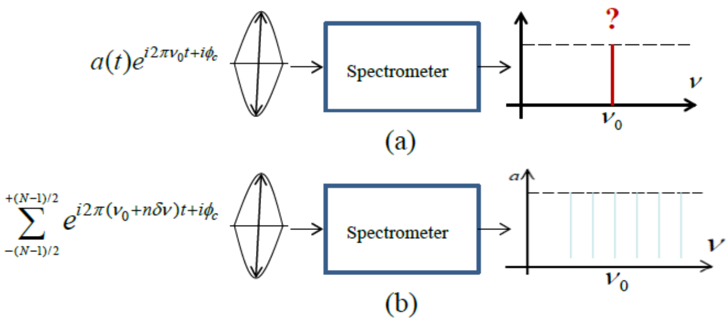
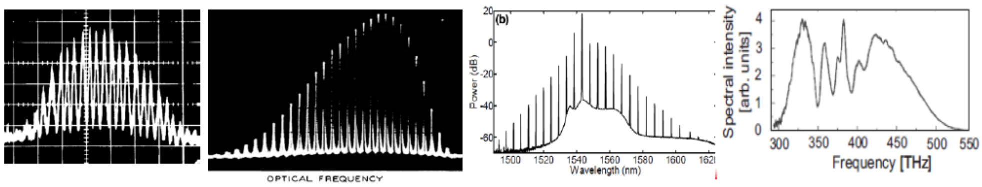


Figure 2. If the mode-locked pulses contain only the central mode frequency, a spectrometric analysis would have given just the single central frequency as in (a), with some additional instrumental fringe broadening. In reality, all the N-frequencies of the comb have been found to be present as in (b) (see also Fig.3).



(a) (b) (c) (d)

Figure 3. Frequency comb (optical spectrum) from mode locked nano second HeNe lasers (a[6],b[7]), from a 300fs micro-cavity ring laser (c[8]) and from ~4fs Ti-Sapphire laser (d[9]). Cavity modes are present in each one of the pulse train because of Non-Interaction of waves (NIW). The spectrum in (d) is very complex, first, because the mode spacing is much smaller than the pulse response function of the spectrometer used and other complexities involved in the measurement (see section 4).

2.2. Presence of frequency comb in the spectral display as Fourier decomposition of pulse-train envelope!

One possible explanation of the presences of the frequency comb is the inverse of the time-frequency Fourier synthesis, which is an assumption that spectrometers are capable of Fourier decomposing a train of periodic pulse envelope into mathematical frequencies whose sum can reconstruct the pulses. This is equivalent to inverting the steps of Eq.2, as shown below (Fig.4):

$$\sum_{-(N-1)/2}^{+(N-1)/2} a(t-n\tau)e^{i2\pi\nu_0 t+i\phi_c} = \frac{\sin N\pi(t/\tau)}{\sin\pi(t/\tau)}e^{i2\pi\nu_0 t+i\phi_c} = e^{i2\pi\nu_0 t+i\phi_c}\sum_{-(N-1)/2}^{+(N-1)/2} e^{i2\pi(n\delta\nu)t} = \sum_{-(N-1)/2}^{+(N-1)/2} e^{i2\pi(\nu_0+n\delta\nu)t+i\phi_c} \quad (3)$$

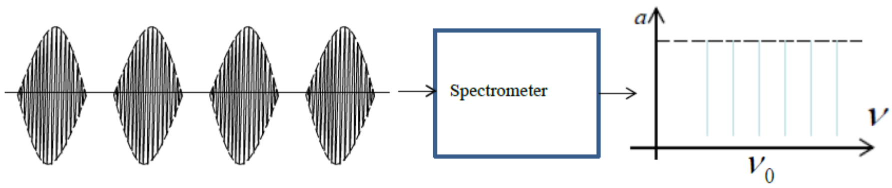


Figure 4. Fourier decomposition of a periodic train of pulses with unique single carrier frequency is equivalent to the frequency comb the laser cavity started with. This is an impractical and a non-causal assumption because even a single pulse can display the frequency comb if the mode spacing is sufficiently large[8] as in micro ring cavity lasers (Fig.3c above).

While Eq.3 is self consistent with mathematical logics, it is impossible in the real world when we consider the required *physical processes*. Fourier decomposition requires the responding instrument (i) first, to read the entire train of pulses, (ii) second, to store the amplitude information in its memory, and then (iii) third, to apply the Fourier decomposition algorithm! A grating in a spectrometer is a linear device that simply splits the incident pulse amplitude into N-new periodically delayed pulse train. It is incapable of carrying the Fourier *decomposition process* since it does not have the capability to physically carry out any of the above there steps. See Section-4 on spectrometric theory for generic pulsed light.

2.3. Fourier synthesis of EM waves by free electron as a broad band detector

However, Fourier synthesis-like process can be carried out by physical devices like electronic instruments designed to carry out the necessary steps. There are also natural detectors that can carry out the steps to a limited degree. There is a unique experiment carried out by Krausz's group[9], where they have traced out the strength of the resultant E-vector in their fs pulse and obtained a unique single carrier frequency under the fs pulse envelope resembling the Fourier synthesis as in Eq.2 (see Fig.5).

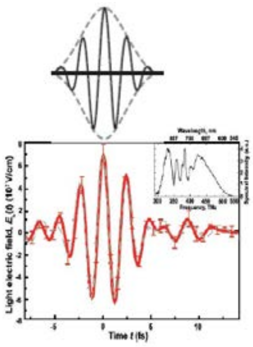


Figure 5. Strength of the resultant electric vector under a few fs Ti-Sapphire pulse, as analyzed by unique step-by-step atto second resolved photo ionization process. It resembles the presence of a single central mode frequency. The trace represents the square root of the accelerated energy of the photo electrons under the influence of separate individual fs pulses, each relatively delayed from the other by some atto seconds[9].

We are in basic agreement with the analysis of this result presented by Krausz's group[9], where the absorbed energy by the photo ionized free electron is represented by the square of the sum of the real amplitudes of all the cavity modes (present in the pulse), which give rise to a sinusoidal trace of the photo electron acceleration, as in Eq.4. The key point here again is the physical process. A free electron is capable of responding to almost the entire range of EM wave frequencies. So its amplitude stimulation becomes proportional to the sum of all the incident field amplitudes. But, it absorbs energy from the superposed fields proportional to the square of the resultant amplitude. Thus, a free electron literally carries out the instantaneous summation of all the simultaneous field induced stimulations, which mathematically resembles Fourier's amplitude summation. Superposition effect is always carried out by a detector that has broad response characteristics. It is a brilliant experimental implementation of measuring very fast resultant E-field summation. This approach will also trace out the E-vector oscillation strength when there is only a single carrier frequency in the pulse.

$$E(t) \sim \left[\mathrm{Re} \sum_{-(N-1)/2}^{+(N-1)/2} e^{i2\pi(\nu_0 + n\delta\nu)t + i\phi_c} \right]^2 = \left[a(t - n\tau)\cos(2\pi\nu_0 t + \phi_c) \right]^2 \quad (4)$$

3. MODELING MODE-LOCKING AS A TIME-GATING OPERATION

We have already mentioned that waves do not interact with each other to create the pulse train out of a mode-locked laser. The intra-cavity phase locker (saturable absorber, Kerr lens modulator, etc.) becomes a time-gating device resonant with the cavity round trip period. The cavity *dumps* its energy in resonance with the opened output gate. The evolution of this resonant time-gating process is a synergistic cooperation between the cavity gain mechanism (stimulated emission) and the cavity field induced transparency of the time-gating device in front of the output mirror. Traditionally, modeling is done by starting with a CW spontaneous emission and propagating this field through the time-gating device back and forth while adding stimulated gain in every pass, until the equilibrium state is attained when the gating device opens up fully in resonance with the round trip cavity time. Before we propose our modeling approach to phase locked cavity pulsing, we need to define and justify the use of the starting wave packets.

3.1. Model for spontaneous and stimulated photon

Our assertion is that in the energy conserving causal world, a real CW radiation can exist only if there is a continuously driven oscillator having a continuous supply of energy to refurbish its radiative loss, like, a radio signal transmitter. In general, Fourier monochromatic modes spanning over all space and time, should not be used as a starting platform to model light-matter interaction phenomena. This practice should be avoided even though it has been *working* for a couple of centuries. The successes obtained so far redirects our attention from mapping/modeling *physical processes* to simply *modeling measurable data*. Atoms and molecules are quantized devices. All their emission and absorption processes are discrete and they must go through their quantum cycles taking a finite time. In fact, this cycling time of lasing atoms or molecules is a key parameter in choosing them for generating high energy and ultra short pulses. It is accepted that a laser oscillation starts from spontaneous emission, which is then amplified through stimulated emission using the cavity feedback mechanism. Both of these spontaneous and the stimulated emission from a given atom consist of discrete packets of energy $h\nu_{mn}$; m and n denoting the quantum transition levels. The classical hypothesis of Huygens-Fresnel describes with remarkable accuracy as to how the wave fronts evolve through diffraction, whether the propagation is taking place through cosmic distances, or though laser cavities of macro or nano dimensions. Then it makes rational sense to propose a classical physical wave model for the photons, whether emitted through the spontaneous or the stimulated process. Our proposal is for a super-exponential wave packet, which rises extremely fast at its front end and dies out exponentially[10,11] [Fig.6]. The model preserves the predictions of QM: The carrier frequency is ν_{mn} ; The total energy under the wave packet envelope is $\Delta E_{mn} = h\nu_{mn}$; the spectral line width of this classical "photon" with super exponential pulse envelope is very close to Lorentzian. Section 4 gives the causal response function of a classical spectrometer to an arbitrary pulse envelope.

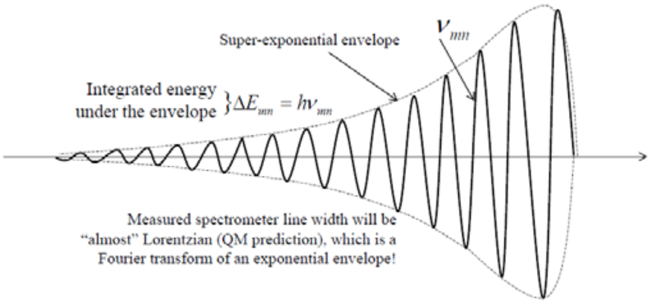


Figure 6. Proposed classical model for photons generated through spontaneous and stimulated emissions. It is a super-exponential wave packet that conforms with most of the requirement of classical and quantum physics.

3.2. Modeling the evolution of resonant time-gating operation

Our proposal is to start propagating several super-exponential pulses from random position with different frequencies within the cavity ensuring their randomness in phase. Then the starting light amplitudes that would stimulate the phase-locker are:

$$\underrightarrow{i_1(t)} = \sum_m \sum_n a(t-t_n) \cdot \exp[i2\pi\nu_m(t-t_n)+\varphi_m] \tag{5a}$$

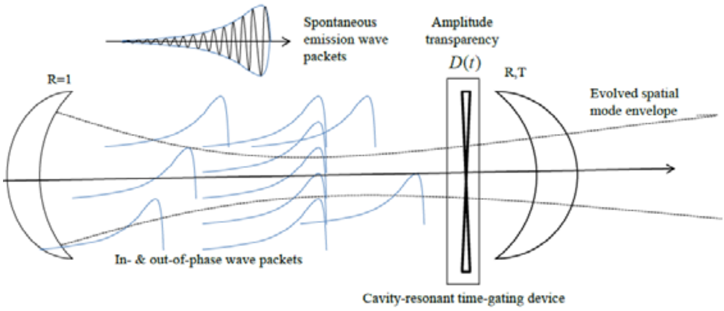


Figure 7. Proposed steps for modeling the evolution of pulsed cavity modes and resonant time-gating operation of the phase locker. One needs to propagate super-exponential pulses starting with random spontaneous emission that is eventually dominated by wave packets due to stimulated emission. Resonant time-gating oscillation becomes automatically synchronous with the cavity round trip time since the contrast of its gate opening (depth of amplitude modulation) is enhanced by the resultant intensity it experiences after each round trip.

Amplitude stimulation of the phase-locker will be given by the Eq.5a multiplied by the first order polarizability ${}_1\chi$ of the materials in the phase-locker, when it is a traditional saturable absorber:

$$d_1(t) = \sum_m \sum_n {}_1\chi a(t - t_n) \cdot \exp[i2\pi\nu_m(t - t_n) + \varphi_m] \tag{5b}$$

Then, the amplitude transparency of the saturable absorber to intra-cavity field will be proportional to the energy it absorbs from the composite field, which is the square modulus of Eq.5b

$$D_1(t) = \left|\sum_m \sum_n {}_1\chi a(t - t_n) \cdot \exp[i2\pi\nu_m(t - t_n) + \varphi_m]\right|^2 \tag{5c}$$

For nonlinear devices like Kerr lens modulator (KLM), the appropriate n-th order polarizability factor ${}_n\chi$ should be used in Eq.5b along with raising the sum total real E-field to that n-th power in Eq.5b. This immediately tells us why KLM is a far superior choice for generating fs pulses than saturable absorber. A saturable absorber has to undergo a pair of time-taking quantum transitions (absorption/relaxation) before it returns to its cavity blocking state. The *classical* nonlinear index changing property of KLM is almost instantaneous simply due to the presence of the composite field without the need of any time consuming energy exchange process. Assuming that the gate is very close to the output mirror, the re-entrant amplitude is the first outgoing amplitude $\underrightarrow{i_1(t)}$ of Eq.5a modulated by the time-gate $D_1(t)$, which is then multiplied by T (twice multiplied by the amplitude transparency of the output mirror):

$$\underleftarrow{i_2(t)} = \underrightarrow{i_1(t)} D_1(t) T \tag{6}$$

This re-entrant signal then picks up many in-phase stimulated pulses through its cavity round trip. Heuristically we assume that the entrant signal got multiplied by the round trip gain $\eta_g \underrightarrow{i_2(t)}$. In reality, there is most likely a finite ultra short time delay between the stimulating pulse and the pulse emitted through stimulated emission as it constitutes a *quantum process*, even though they are in phase. However, we are neglecting this subtle effect at this proposal state of modeling. Then the new outgoing amplitude, along with some more spontaneous emissions, is given by:

$$\underrightarrow{i_3(t)} = \eta_g \underrightarrow{i_2(t)} + \sum \text{spontan. emsns.} \tag{7a}$$

The corresponding amplitude stimulation experienced by the saturable absorber, is given by Eq.7b. The consequent energy absorption is given by Eq.7c, which is also the amplitude modulation factor for the outgoing signal.

$$d_3(t) = \chi \underrightarrow{i_3(t)} \tag{7b}$$

$$D_3(t) = |d_3(t)|^2 \tag{7c}$$

The next re-entrant amplitude through the time-gating device is then:

$$\underleftarrow{i_4(t)} = \underrightarrow{i_3(t)} D_3(t) T \tag{8}$$

Such an iterative process should be continued until the steady dynamic state is achieved. We are in the process of implementing such a modeling algorithm and the results will be presented in the future.

4. MODELING SPECTROMETER WITH PHOTON WAVE PACKETS

The modeling approach proposed above using photon as a classical wave packet (Fig.6) is supported by our formulation of spctrometry[12,13]. An N-slit grating spectrometer is a linear replicator of the incident pulse into N-new pulses with a periodic step delay of $\tau = m\lambda / c$ at the m-th order (Fig.8). It immediately gives us a new practical parameter for spectrometers that we have never recognized before, which is the spectrometer time constant, $\tau_0 = N\tau = Nm\lambda / c = R\lambda / c$, where R is the classical resolving power of the spectrometer. So, in reality, the spectrometric fringes are always time varying, if we can always use very fast detector, except when we are analyzing high intensity spontaneous emission consisting of innumerable super-exponential pulses so randomly distributed that our detector always experiences a steady average intensity. Let us develop formulation for a single photon wave packet. It is the detecting dipole that sums the joint stimulation it experiences due to N-waves superposed on it. So, the total amplitude stimulation experienced by the detector with first order polarizability ${}_1\chi$ placed at a delayed-location τ would be given by the Eq.9.

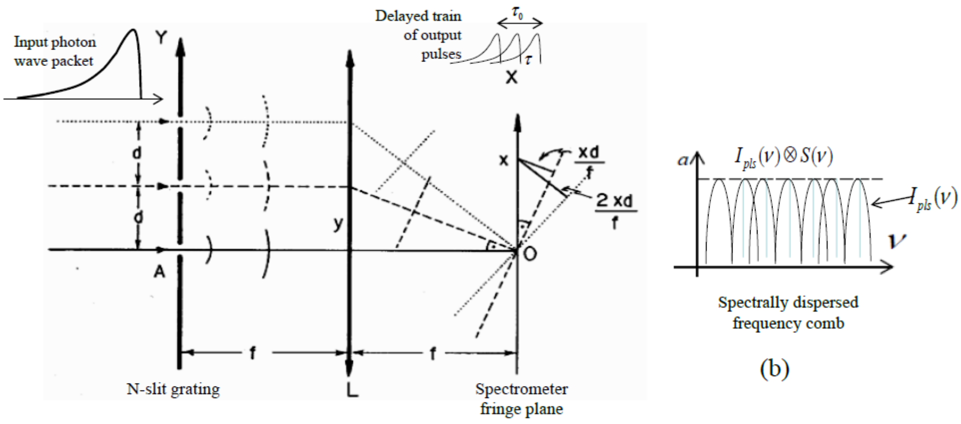


Figure 8. (a) Modeling a spectrometer based on physical process by propagating a photon wave packet and assuming that an N-slit grating produces N-periodically delayed and partially superposed pulses that create time varying intensity on a detector. During propagation, from grating to the detector, the replicated pulses remain unaltered due to each others' presence, since waves do not interact with each other. (b) If one has n-identical pulses with n-different frequencies, each carrier frequency will appear in its proper location determined by the grating "order equation"; but the fringes will be broadened due to partial overlap of their own N-replicated pulses. This broadening does not represent generation of new optical frequencies by the grating.

$$i_{out}(t) = \sum_{n=0}^{N-1} ({}_1\chi / N) a(t - n\tau) \cdot \exp[i2\pi\nu(t - n\tau)] \tag{9}$$

The time varying intensity is the square modulus of Eq.9:

$$|i_{out}(t)|^2 = \left| \sum_{n=0}^{N-1} ({}_1\chi / N) a(t - n\tau) \cdot \exp[i2\pi\nu(t - n\tau)] \right|^2 \tag{10}$$

If we use a time integrating photographic or a CCD camera, then integration of Eq.10 and algebraic simplification[13] will give us the pulse response function for a specific frequency ν at a τ -delay location:

$$I_{pls}(\nu, \tau)\Big| = \frac{{}_1\chi^2}{N} + \frac{2\,{}_1\chi^2}{N^2} \sum_{p=1}^{N-1} (N - p)\gamma(p\tau) \cos[2\pi p\nu\tau] \tag{11}$$

The pair-wise autocorrelation $\gamma(p\tau)$ is given by Eq.12, where $p = |m - n|$; m and n assuming all allowed values of N.

$$\gamma(p\tau) = \int d(t - n\tau) d(t - m\tau)\, dt \Big/ \int d^2(t)\, dt \tag{12}$$

The significance of the spectrometer time constant τ_0 derived by focusing on causal *physical process* will now be apparent. For an incident pulse, which is longer than τ_0, the time integrated fringe width of Eq.11 becomes identical to the classical grating formula derived by using a Fourier monochromatic mode existing in all time (CW), except the detector's polarizability factor. The physical meaning is that the correlation factor $\gamma(p\tau)$ approaches unity when the incident pulse is longer than the spectrometer time constant τ_0.

$$\underset{\delta t \to \tau_0 = N\tau}{Lt.} I_{pls}(\nu, \tau) = \frac{{}_1\chi^2}{N} + \frac{2\,{}_1\chi^2}{N^2} \sum_{p=1}^{N-1} (N - p) \cos[2\pi p\nu\tau] \equiv \frac{{}_1\chi^2}{N^2} \frac{\sin^2 \pi N\nu\tau}{\sin^2 \pi\nu\tau} \equiv I_{cw}(\nu, \tau) \tag{13}$$

In the early days of low resolution spectrometry, given the length of the spontaneous emission photons on the order of one or a few nano seconds, the $(\sin^2/\sin^2)$ formulation was quite accurate. Note, further that by using the Parseval's energy conservation theorem (integrated energy under the pulse), one can show[13] that our pulse response function $I_{pls}(\nu,\tau)$ is given by Eq.14, in case one is using a pulse shorter than τ_0. This has been the classical assumption that a short pulse produces spectral broadening that can be quantitatively explained by the presence of Fourier frequencies due to the pulse envelope, $I_{cw}(\nu)\otimes\tilde{A}(\nu)$, where $I_{cw}(\nu)$ is the classical CW-grating function and $\tilde{A}(\nu)$ is the Fourier intensity spectrum derived from the amplitude envelope $a(t)$.

$$I_{pls}(\nu,\tau)\approx\int_{-\infty}^{\infty}\left|\,i_{out}(t)\right|^2 dt = I_{cw}(\nu)\otimes\tilde{A}(\nu) \qquad (14)$$

Then, for a frequency comb, the optical spectrum will be $I_{pls}(\nu)\otimes S(\nu)$ (see Fig.8b), where $S(\nu)$ has been given in Eq.1.

5. DISCUSSIONS

Consistent engineering innovations require understanding the real physical interaction processes in an optical phenomenon that we can emulate to create new device, tool or technology[1]. The prevailing scientific trend has been modeling the measurable data without careful attention to the interaction processes. Here we have demonstrated that the physical process behind mode-lock pulse generation is not simple summation of the laser cavity modes. This is because EM fields do not interact with each other to create new distribution of energy as space and time domain pulses[2,14]. Time-gating property of the intra-cavity phase-locker plays more critical role than we tend to appreciate. Frequency comb could not have existed if mode-locking was really due to Fourier summation of the cavity modes, which mathematically converts all the cavity frequencies into a new mean central frequency. By not focusing on the physical processes behind natural phenomena, we have missed recognizing that waves, in general do not interact with each other in the linear domain. Consequently, the time-frequency Fourier theorem, representing linear summation of harmonic waves, should be used with great care since it is not a physical principle of nature, even when the measurable data matches with the mathematical construct. If we had recognized that summed modes by themselves do not create the short pulses, fs laser could have been invented a couple of decades earlier. Inventors would have given primary attention to identify fastest possible time-gating phenomenon, like Kerr lens modulator, and secondary importance to broadest possible spectral band width of the lasing material, which is necessary for shortest possible recycling time of the lasing atoms to extract high energy pulses. The third key point of this paper is that optical phenomena should be formulated based upon propagating a space and time finite pulses. Finite pulses conserve energy and hence they are causal signals and are more appropriate for modeling classical optical instruments. We have proposed that we should use a classical super-exponential wave packet as the model for both stimulated and spontaneous emission[10,11]. As a case example, we have given the summary of the formulation for a classical spectrometer[12,13] and have demonstrated that this new formulation represents a better physical model for analyzing femto second laser comb.

A working theory consistently validating measured data does not imply that the theory is the *final theory* for us. We must test the capability of working theories to guide us in visualizing the invisible interaction processes that give rise to the measurable data. When this test fails, we must re-frame the foundational hypotheses and re-derive the theories. Our identification of the property, non-interaction of waves (NIW), and its application in re-formulating the phase-locked laser pulsing theory, is a relevant example in support of our Interaction Process Mapping Epistemology (IPM-E). IPM-E must always be applied in conjunction with Measurable Data Modeling Epistemology (MDM-E). IPM-E cannot replace MDM-E. Such an approach makes invention process (emulation of natural phenomenon to create new technologies) more efficient.

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