SPIE Conf. Proc. Vol.**9570**-9 (2015)

DOI:10.1117/12.2188291

Did Planck, Einstein, Bose count indivisible photons, or discrete emission / absorption processes in a black-body cavity?

Michael Ambroselli, Chandrasekhar Roychoudhuri

Physics Department, University of Connecticut, Storrs, CT 06269

ABSTRACT

Planck, Einstein and Bose all had to introduce statistics, and thus *counting*, in order to successfully derive an equation for the energy distribution within the black-body radiation spectrum, and what we now call Bose-Einstein statistics. Some of the *details* involved in the counting procedure vary while still giving the same result. However, the interpretation of *what* we count may differ dramatically from one another (as, for example, between Planck and Bose), without impacting the final, mathematical result. We demonstrate here a further alternative, which varies both, in the details of the counting, as well as in the interpretation, while still producing the same well known statistics. This approach puts the "quantumness" back into the radiation emission/absorption process, possibly dispensing with the requirement of quantized light, at least in the context of black-body radiation.

Keywords: black body, black-body radiation, bose-einstein statistics, photons, quantization

1. INTRODUCTION

In 1901 Planck used the proposal of quantized energy exchange between material oscillators of, and the radiation field within a black-body cavity to derive his well-known equation for the corresponding radiation energy density.[1] Later, in 1906, Einstein suggested that Planck actually introduced quantization of light itself.[2] Planck, however, continued to question the validity of Einstein's light-quantum hypothesis.[3]

Some sources still claim the necessity for quantized light in order to explain phenomena such as the photoelectric effect[4] and the black-body spectrum.[4,5] Scully and Lamb have shown that the former finds a valid mathematical description using a semi-classical approach via Quantum Mechanics (QM) and classical non-quantized light.[5] Planck's argument that lead to the mathematically correct description of the black-body spectrum only required quantized material oscillators (atoms / molecules), and we will here add to this line of reasoning.

Most modern derivations of Planck's formula seem to prefer the more mathematical and less physical route of plain combinatorics over one rooted in physical considerations.[6,7] We will summarize the approaches taken by Planck, Einstein and Bose in deriving the quantum statistics involved in the black-body description, and present an additional way of arriving at the same result.

2. HISTORICAL SUMMARY

All three authors required two key elements in order to successfully derive the energy distribution of black-body radiation: the use of *statistics* and energy-quantization. Each approach differed dramatically from the others, but reached the same final result. We will later see why this might have come about.

2.1 Planck

In light of the failure of the Rayleigh-Jeans law in the UV, and that of Wien's law in the IR, Planck struggled for a while[8–10] to find a mathematically accurate description of the black-body energy distribution valid for *all* frequencies. When he finally succeeded,[1,11] he had taken a vital, pioneering step towards QM as we know it today.

Planck had realized, that he needed to find a connection between the second law of thermodynamics and electromagnetic theory.[9] To do this, he needed to come up with an entropy function, $S_N = k \log \mathfrak{R}$, for the black-body system, which in turn required him to *count* something (i.e. the number of micro states $\mathfrak{R}$ of the system). Thus, Planck *postulated* that the mean energy U_N of all N oscillators that make up the system can only take on integer (P) multiples of some unspecified energy element ε, i.e.

$$U_N = P \cdot \varepsilon \tag{1}$$

Now he could count the number $\mathfrak{R} = \frac{(N+P-1)!}{(N-1)!P!}$ of micro states, each of which he calls a "complexion", that make up a given macro state of total energy U_N, and hence write down an entropy. In order to progress further, Planck now had to introduce Wien's displacement law, $E \cdot d\lambda = \vartheta^5 \psi(\lambda\vartheta) \cdot d\lambda$, where ϑ stands for the temperature. He rewrites this in terms of *frequency*, before actually using it*.

With this, he reaches the conclusion that $\epsilon = h\nu$ and finds the final form of the energy distribution formula

$$u = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{k\vartheta}} - 1} \tag{2}$$

2.2 Einstein

Einstein introduces statistics into his derivation of the black-body equation via his A and B coefficients, which presuppose the quantized nature of the emitters / absorbers (atoms / molecules).[13] His treatment also already *assumes* that the system has reached thermal equilibrium. Thus, he does not require the variational procedure Bose went through, as we will see, in order to arrive at his version of Planck's equation

$$B_m^n \rho = \frac{A_m^n}{e^{\frac{\varepsilon_m - \varepsilon_n}{kT}} - 1} \tag{3}$$

The main physical input to derive this expression came from the requirement of a balance between the processes of spontaneous and stimulated emission, and absorption (i.e. thermal equilibrium between the emitters and the radiation).

The three coefficients (A_m^n, B_m^n and B_n^m, for spontaneous emission, stimulated emission and absorption respectively) thus do not represent individual properties of a given kind of atom or molecule, but rather a *statistical, average* description of a large ensemble of these particles, which hides, in a sense, one or many individual physical processes.

To recover the full form of the equation, Eq.(2), Einstein, like Planck, needed to use Wien's displacement law.

2.3 Bose

Some years later, in 1923, Bose suggested that one did not need, and in fact should not use, the (classical) displacement law to derive the black-body result.[14] He criticized both, Planck's as well as Einstein's derivations, for lacking logical justifications, by which he mainly referred to their reliance on classical assumptions. To provide a derivation that removes these apparent shortcomings, he picked as his starting point the *assumption*

*In his 1906 book, he points out the more fundamental character of the frequency parameter[12] as opposed to wavelength, which changes from medium to medium, and thus represents a *derived quantity.* This underscores an important physical consideration that to this day remains widely under-appreciated.

of quantized light (Einstein's "indivisible quanta"), which together with the formalism of statistical mechanics provided everything he needed.

In order to apply statistical mechanics, Bose had to somehow distribute his light quanta within a given closed volume V in a well defined way. To do this, he, quite arbitrarily as he himself admitted,[14] divided the associated phase space up into cells of size h^3, where h corresponds to Planck's constant. However, since the reason that h occurs in his equations in the first place at most comes down to an arbitrary scale / unit convention for the action, we cannot, and *should* not, interpret this manner of division as in any way physically significant. Ralston gives a very interesting account of the generality of this idea.[15–17]

Through this division of the phase space of his light quanta, Bose finds the total number of cells associated with a given "specie"† s of quanta as

$$A^s = V \frac{8\pi\nu^2}{c^3} d\nu^s \tag{4}$$

Then, defining p_r^s as the number of cells of the s-variety that contain r quanta, he gets for the total number of all possible permutations of available light quanta

$$\prod_s \frac{A^s!}{\prod_r p_r^s!} \tag{5}$$

From the definition of p_r^s we also have that

$$N^s = \sum_r r p_r^s \qquad ; \qquad A^s = \sum_r p_r^s \tag{6}$$

The logarithm of Eq.(5), under the condition that its variation with respect to p_r^s vanishes, given the following constraint,

$$E = \sum_s N^s h\nu^s \tag{7}$$

and those of Eqs.(6), results in the entropy of the system. He now arrives at his version of the statistics later named for him

$$N^s = \frac{A^s \exp(-h\nu^s/\beta)}{1 - \exp(-h\nu^s/\beta)} \tag{8}$$

and after some algebra, putting everything together yields Planck's black-body equation.

3. ALTERNATIVE DERIVATION

Planck, Einstein and Bose all made certain assumptions, in addition to physically observed facts, in order to derive their results. Particularly in Bose's case the physical implications of these assumptions seem somewhat opaque and have caused, in our opinion, much confusion.

We start with the experimentally validated assertion, that atoms and have sharp energy levels and can only absorb or emit energy (light) in quantities matching the differences between such levels. Molecules and higher order composites of atoms and molecules will have increasingly more complicated level structures that will eventually turn into continua (bands). So what we will count will essentially amount to the number of levels corresponding to a given energy in such a system.

Now, let us imagine that our body has a total of N_ν available energy levels corresponding to energy ϵ_ν. Of these, n_ν have absorbed a corresponding amount of energy. Thus, we have for the total amount of energy contained

---

†The members of each specie essentially correspond to the photons of a given frequency.

$$E = \sum_\nu n_\nu \epsilon_\nu \tag{9}$$

where ϵ_ν represents the energy of a transition corresponding to frequency ν.

We now want to find the number of excited levels of a given energy at thermal equilibrium. To do this, we need an expression for the entropy S of the body and then find where S has an extremum. This corresponds to the condition $\delta S = 0$. There exist $\binom{n_\nu + N_\nu - 1}{n_\nu}$ ways to distribute n_ν amounts of energy ϵ_ν among N_ν corresponding states.

To find the *total* number Ω of all possible states, we multiply all these together

$$\Omega = \prod_\nu \binom{n_\nu + N_\nu - 1}{n_\nu}, \tag{10}$$

which results in an entropy (after using Stirling's approximation for large n_ν, and dropping the -1‡) of

$$\begin{aligned} \frac{S}{k_B} &= \ln \Omega \\ &\approx \sum_\nu (n_\nu + N_\nu) \ln (n_\nu + N_\nu) - n_\nu \ln n_\nu + n_\nu - \ln(N_\nu!) \end{aligned} \tag{11}$$

Setting the variation of this with respect to n_ν§, under the constraint Eq.(9), equal to zero in order to find the equilibrium condition results in

$$\ln (n_\nu + N_\nu) - \ln n_\nu - \beta \epsilon_\nu = 0 \tag{12}$$

and we end up with

$$n_\nu = \frac{N_\nu}{e^{\beta \epsilon_\nu} - 1} \tag{13}$$

which matches Eq.(8). Note, however, that this represents / describes the energy distribution within the material object under consideration, not the radiation within a cavity, and thus demonstrates more clearly that this result may apply to any material object in thermodynamic equilibrium (if it satisfies the physical assumptions made).

4. DISCUSSION

All discussions regarding the radiation from a black body *necessarily* made and make use of statistical mechanics, due to the large quantity of matter that actually produces the described spectrum. We know that single atoms and molecules do not produce anything close to a black-body spectrum. Rather, we see line spectra, which tell us about the quantized nature of the energy levels of *atoms and molecules*. Hence we argue, that we cannot possibly draw any conclusions about the various details of the individual elementary processes involved in the emission or absorption of radiation based on simple measurements of spectra or the photo-electric effect. The relevant equations only model the final measurable energies, not the physical interaction processes that produce them. From this we can also conclude that only macroscopically large systems have the ability to produce anything close to black-body radiation. Sufficiently small systems¶ will not exhibit this phenomenon.

The successful derivation of Planck's black-body equation also requires the introduction of discrete energies $E_{mn} = h\nu_{mn}$. Planck's reason for doing this originated mainly with the need to count energy states in order

‡Which we can legitimately do when Stirling's approximation applies

§The N_ν always remain constant for a given object, so we do not consider their variation.

¶As defined by the inapplicability of the Stirling approximation w.r.t. the number of constituent material particles / their energy levels to define an entropy, without debating the physical significance.[1] Einstein uses thermodynamical arguments to show that low energy density monochromatic radiation behaves as what others have called a photon gas.[18] Bose, on the other hand, takes up Einstein's photon proposal, assigning the quantization directly to the light itself and assumes their separate, particle-like existence.[14] Our own derivation in section 3 essentially counts net emissions from quantized energy levels of material particles (atoms). Evidently, as long as we introduce quantized amounts of energy *somehow*, we will arrive at the black-body equation, regardless of what we propose as the physical reason for quantization.

However, as Keller points out,[19] all we know about light we only know through its interaction with matter. Thus, it seems more prudent to interpret the energy quantization inherent to the problem as that of the (discrete) energy levels of the material particles making up the black-body cavity, as we have proposed in section 3, especially since there already exists compelling evidence for the quantized nature of *matter*, rather than taking it as evidence for the quantization of *light* itself.

Acknowledgments

The authors would like to acknowledge and thank Hans de Raedt for reviewing this paper and offering helpful suggestions.

REFERENCES

[1] Planck, M., "Über das Gesetz der Energieverteilung im Normalspektrum," *Ann. Phys.* **309**, 553–563 (Jan. 1901).

[2] Einstein, A., "Zur Theorie der Lichterzeugung und Lichtabsorption," *Ann. Phys.* **325**(6), 199–206 (1906).

[3] Planck, M., "The genesis and present state of development of the quantum theory," *Nobel Lecture* (June 1920).

[4] D'Espagnat, B., [*Conceptual Foundations of Quantum Mechanics*], Westview Press, 2nd ed. (1999).

[5] Lamb Jr., W. E. and Scully, M., "The photoelectric effect without photons," in [*Polarization, Matière et Rayonnement*], **in Honour of A. Kastler**, 363 – 369, Presses Universitaires, Paris (1969).

[6] Griffiths, D. J., [*Intruduction To Quantum Mechanics*], Pearson Prentice Hall, 2 ed. (2005).

[7] Mandl, F., [*Statistical Physics*], John Wiley & Sons Inc., 2nd ed. (2004).

[8] Planck, M., "Entropie und Temperatur strahlender Wärme," *Ann. Phys.* **306**, 719–737 (Mar. 1900).

[9] Planck, M., "Über irreversible Strahlungsvorgänge," *Ann. Phys.* **306**, 69–122 (Nov. 1900).

[10] Planck, M., "Über Eine Verbesserung Der Wienschen Spectralgleichung," *Verh. Dtsch. Phys. Ges.* **2**, 202–204 (Oct. 1900).

[11] Planck, M., "Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum," *Verh. Dtsch. Phys. Ges.* **2**, 237–245 (Dec. 1900).

[12] Planck, M., [*Vorlesungen Über die Theorie der Wärmestrahlung*], Verlag von Johann Ambrosius-Barth (1906).

[13] Einstein, A., "Zur Quantentheorie der Strahlung," *Phys. Z.* **18**, 121–128 (1917).

[14] Bose, S. N., "Plancks Gesetz und Lichtquantenhypothese," *Zeitschrift für Physik* (1924).

[15] Ralston, J. P., "Revising your world-view of the fundamental constants," *SPIE Conf. Proc.* **8832**, #41 (2013).

[16] Ralston, J. P., [*Advances in Quantum Mechanics*], ch. 19 (Emergent un-Quantum Mechanics), 437–476, InTech (Apr. 2013).

[17] Ralston, J. P., "Quantum Theory without Planck's Constant," *ArXiv e-prints* **arXiv:1203.5557 [hep-ph]** (Mar. 2012).

[18] Einstein, A., "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt," *Ann. Phys.* **17 (8)**, 132–148 (1905).

[19] Keller, O., [*Light - The Physics of the Photon*], Series in Optics and Optoelectronics, CRC Press, 1st ed. (June 2014).