Demo.

Demonstrations using a Fabry–Perot. I. Multiple-slit interference

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A method is described by which a multiple-slit interference pattern can be demonstrated quite easily with the help of an inexpensive Fabry–Perot etalon and a low-power laser beam. The separation between the "slits" can be adjusted continuously. The experiment brings forth the similarity between a grating and a Fabry–Perot as a fringe-sharpening device. A simple derivation of the analytical expression for such fringes is presented.

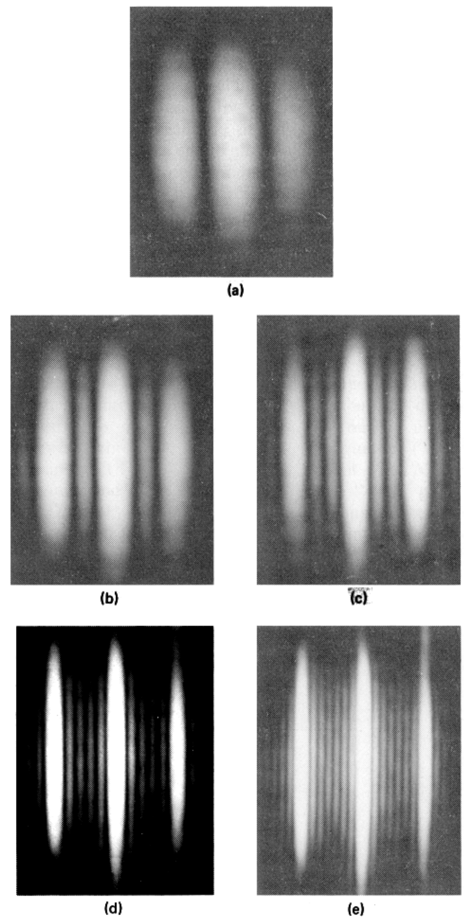

Fig. 2. Multiple-"slit" interference (Fraunhofer diffraction) patterns with a particular slit separation: (a) 2 slits, (b) 3 slits, (c) 4 slits, (d) 6 slits, and (e) 8 slits.

INTRODUCTION

It is a common practice, before introducing the ordinary diffraction grating to the undergraduate students, to describe how the narrowing of fringes takes place in the multiple-slit interference (Fraunhofer diffraction) pattern as the number of slits increases. Reference 1 describes a simple method of demonstrating such an experiment. The method described in this paper is slightly more difficult, but it has more flexibility. In addition, it is capable of illustrating the similarity between a grating and a Fabry–Perot (FP) interferometer in a straightforward manner.

We use a plane-parallel FP interferometer or a FP etalon. The surface of the FP mirrors does not have to be of very high quality since we are not trying to form high-quality FP fringes. (Flats of surface quality $\lambda/10$ or even $\lambda/5$ are good enough for this experiment.) However, the reflectivity should be reasonably high (around 98%), so that the first several beams produced by multiple reflection (those used in the experiment) have energies close to each other. Thus, a pair of $\lambda/5$ flats of about 2-in. diameter with a high-reflection coating is adequate for this purpose.

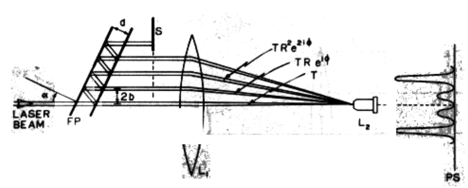

Fig. 1. Experimental setup to demonstrate multiple-slit interference pattern with the help of a Fabry–Perot interferometer and a laser beam. FP, Fabry–Perot; S, screen to control the number of beams; L_1, focusing lens; L_2, microscope objective; PS, multiple-beam fringes on the projection screen.

EXPERIMENTAL SETUP

A narrow light beam straight from a laser (Fig. 1) is incident on one end of the FP etalon at an angle such that the beams generated by multiple reflections are spatially separated from each other as if a multiple-slit screen had been placed in front of an extended collimated beam. At the focal plane of lens L_1, immediately following the FP, one can see interference fringes which have properties similar to the multiple-slit interference pattern. For convenient observation by many people, the fringes can be projected on a screen by a microscope objective (L_2).

The experiment can be conveniently set up in the following way. Lenses L_1 and L_2 are adjusted to be coaxial with the laser (which can be tested by observing the beams reflected from the lens surfaces). Then the FP etalon is inserted at an angle to the axis before L_1, preferably on a turntable, to control the angle of insertion. This control over the angle of insertion will control the separation between the transmitted beams produced by multiple reflection. Thus, a continuous tilt of the etalon will give continuous change in the separation between the "slits." If a two-plate FP interferometer is used instead of an etalon, the parallelism between the plates can be easily achieved by first allowing the incident beam to be perpendicular to the first mirror and then adjusting the other until all the transmitted beams are exactly coincident.

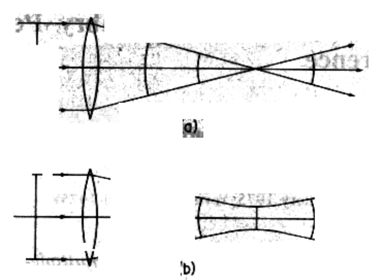

Fig. 4. Focusing of a plane wave by a lens: (a) In an ideal situation an infinite plane wave is focused to a geometrical point. (b) A finite plane wavefront produces a small plane-wave "spot" at the focal plane, whose irradiance distribution is the Fraunhofer diffraction pattern of the finite aperture.

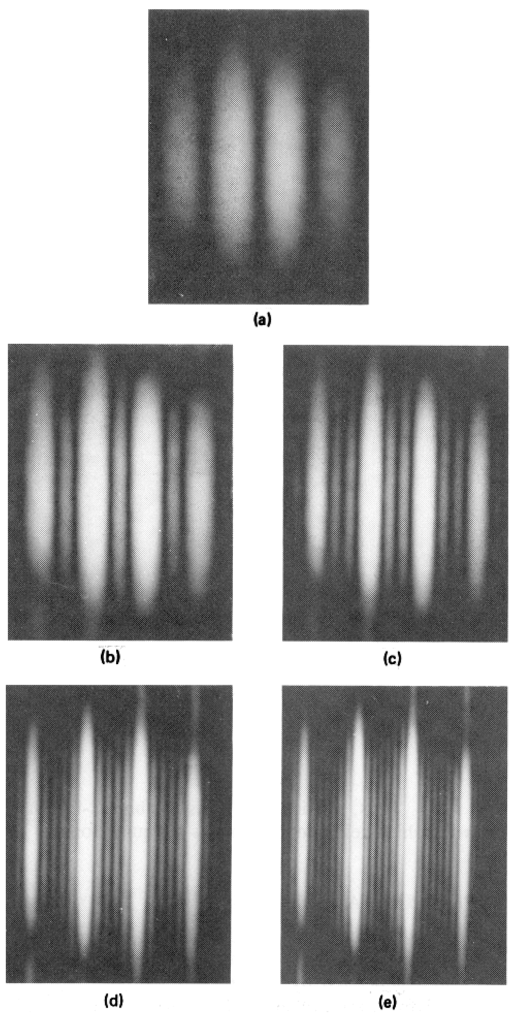

Fig. 3. Multiple-"slit" interference (Fraunhofer diffraction) patterns with a slit separation different from that in Fig. 2: (a) 2 slits, (b) 3 slits, (c) 4 slits, (d) 6 slits, and (e) 8 slits.

The effect of changing the number of slits can now be easily demonstrated by moving a screen S (Fig. 1) to allow as many beams ("slits") as are wanted. Figure 2 shows a series of photographs taken with different positions of this screen. One may recognize the patterns for two, three, four, six, and eight slits with the spacing ($2b$) unchanged in the series. Figure 3 presents a similar series after the angle α—and therefore the spacing, $2b$—has been given a new value.

THEORY

The mathematical expression for such multiple-beam interference with a FP can be derived from its similarity to a diffraction grating. The only difference is the decrease in intensity of the consecutive beams and the increase in path difference due to their traveling through the FP. In Fig. 1, the consecutive beams are shown to have a separation of $2b$ with the appropriate changes in amplitudes and phases. We shall avoid using the Fourier transformation technique by using a simplified model: a plane wave incident on a lens produces a small "spot" of plane wave at the focal plane of the lens (focal spot) where the irradiance distribution follows the Fraunhofer diffraction pattern for a single "slit"—in this case, the lens aperture (see Fig. 4). Then the situation of many plane waves incident on a lens can be visualized in Fig. 5. (The size of the focal spot is exaggerated.) The successive plane waves generated at the focal plane of the lens interfere with each other at angles

$$\theta_n \approx \tan\theta_n \approx \sin\theta_n \approx n(2b/f), \qquad (1)$$

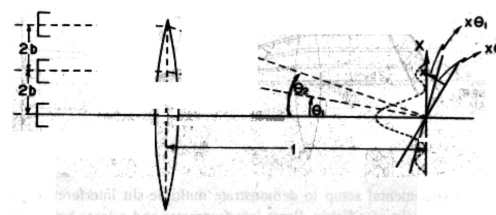

Fig. 5. Multiple-slit Fraunhofer diffraction pattern as interference between plane-wave "spots" with regularly increasing tilt produced by the individual slits.

where n is an integer and f is the focal length of the lens used. So, the phase difference at a point x is

$$\phi_n = kx \sin\theta_n \approx n(2kbx/f) \equiv n\phi, \tag{2}$$

where

$$k = 2\pi/\lambda.$$

Thus, a regular grating equation with the same amplitude (say, unity) and phase from all the slits can be written as[2]

$$[1 + \exp(i\phi) + \exp(i2\phi) + \cdots]$$
$$= \sum_{n=0}^{N-1} \exp(in\phi)$$
$$= \exp\left[i\left(\frac{\phi}{2}\right)(N-1)\right] \frac{\sin(N\phi/2)}{\sin(\phi/2)}, \tag{3}$$

where N is the total number of slits. Then the irradiance distribution is proportional to

$$\frac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}. \tag{4}$$

In our case, the amplitude and phase of the successive beams are modified so that the nth wavefront can be written as

$$TR^n \exp(in\psi), \tag{5}$$

where T and R are the intensity transmittance and reflectance, respectively, and ψ is the phase delay between any two consecutive beams, given by

$$\psi = (2\pi/\lambda)(2\,d\rho_\lambda \cos\alpha), \tag{6}$$

where d and α are shown in Fig. 1, and ρ_λ is the refractive index of the medium between the two reflecting surfaces. We shall neglect any phase shift in the reflecting surface.

So, the resultant amplitude of interference for our case is

$$A = \sum_{n=0}^{N-1} TR^n \exp(in\psi)\exp(in\phi) \tag{7}$$

instead of (3), and the irradiance distribution is

$$I = \mathcal{T} \frac{1 + F_N \sin^2[N(\psi/2 + \phi/2)]}{1 + F \sin^2(\psi/2 + \phi/2)} \tag{8}$$

instead of (4), where

$$\mathcal{T} \equiv T^2(1 - R^N)^2/(1 - R)^2, \tag{9}$$

$$F_N \equiv 4R^N/(1 - R^N)^2, \tag{10}$$

and

$$F \equiv 4R/(1 - R)^2. \tag{11}$$

Equation (8) will also describe the fringe pattern for a Lummer–Gehrcke plate.[3] This suggests that the demonstration experiment can also be carried out by using a Lummer–Gehrcke plate, if available. Equation (7) will give an exact FP Airy function in the limit when N tends to infinity and the separation between the "slits" goes to zero ($2b = 0$; i.e., $\phi = 0$). The same equation becomes identical with the ordinary diffraction grating equation (4) when R tends to unity and ψ is zero.

CONCLUSION

We have described a simple method of demonstrating fringe sharpening due to multiple-slit interference by using an inexpensive Fabry–Perot etalon. The theory has been developed in such a way as to emphasize the similarity between a diffraction grating and a Fabry–Perot in achieving fringe sharpening by multiple-beam interference.

ACKNOWLEDGMENTS

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[1] V. B. Elings, Am. J. Phys. **38**, 1263 (1970).

[2] F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill, New York, 1957), p. 330.

[3] M. Born and E. Wolf, *Principles of Optics* (Pergamon, London, 1970), pp. 341–344.