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IS FOURIER DECOMPOSITION TECHNIQUE APPLICABLE TO INTERFERENCE SPECTROSCOPY?

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SUMARIO

Si los patrones de haces múltiples ocurren solamente cuando los haces se han superpuesto en sentido real físico, entonces la función de respuesta de instrumentos como el Fabry-Perot y la rejilla a pulsos cortos debe ser dependiente a la forma y orden de la interferencia. La conclusión anterior no sigue de la técnica de descomposición de Fourier. Además, encontramos que la frecuencia instantánea es más relevante que las componentes en frecuencia de Fourier en experimentos de interferencia.

ABSTRACT

If multiple-beam interference pattern is produced only when the multiple beams are superposed in the real physical sense, then the instrumental response curve of Fabry-Perots and gratings to short pulses should be pulse-shape and order dependent. Such a conclusion does not conform with the Fourier decomposition technique. Further we find that the instantaneous frequency is more relevant than Fourier component frequency in interference experiments.

I. *Introduction*

In a previous series of papers (Roychoudhuri 1975a, 1975b, 1976, 1977) an attempt was made to analyze the response of Fabry-Perot interferometers (FP) and gratings to light pulses of very short duration using the causal concept of real physical superposition of the multitude of beams replicated from the parent pulse by these instruments. The major conclusion has been that the instrumental response function for conventional spectrometers like FP and gratings depends not only on the pulse shape but also on time, which has apparently been ignored by people doing spectroscopy of pico- and nanosecond light pulses (both regular coherent and random spontaneous pulses). Thus to do proper spectroscopy of pulsed light, one must know the shape of the incident pulse to compute the instrumental response curve (see Equation 14) that should be deconvolved from the recorded instrumental pattern to extract the true spectral information.

Users of Fourier transform technique of decomposing a time-pulse into a series of infinitely long (monochromatic) single frequency radiations apparently feel no need for such a conclusion, because they can deconvolve the monochromatic instrumental response function from the recorded pattern due to a pulse.

So in the next section we shall compare the results of Fourier decomposition technique with those obtained by the principle of real physical superposition. And following that we shall present a discussion on the reality of Fourier component frequencies.

II. *Comparison of the Two Interpretations*

Let us begin with the Fourier decomposition interpretation. Suppose we clip off an ideal rectangular pulse $f(t)$ of width δt (Figure 1) from a stabilized single mode (ν_0) laser beam focused through a, say 10 μm, pinhole on the rim of a disc rotating at a very high speed. Then the Fourier decomposition implies that

$$f(t)\ [\text{rect. of width } \delta t] \xleftrightarrow{F.\,T.} F(\nu - \nu_0) = \delta t \operatorname{sinc} \pi (\nu - \nu_0)\, \delta t \qquad (1)$$

Thus we have a large number of monochromatic frequencies given by a sinc function whose full width at half the maximum (Figure 1) is $\delta\nu_F$ where

$$\delta\nu_F \delta t = \overset{1.2}{\cancel{0.9}} \qquad (2)$$

Let us now imagine that we have two gratings of the same resolving power $\mathscr{R} = 10^5$: the first one with a total number of lines $N = 10^5$ working at the $m = 1$ order and the second one with $N = 10^3$ lines and $m = 10^2$ order. Classically, the resolving power is

$$\mathscr{R} = \frac{\nu}{(\delta\nu_G/\underset{1.2}{\cancel{0.9}})} = mN = 10^5, \qquad (3)$$

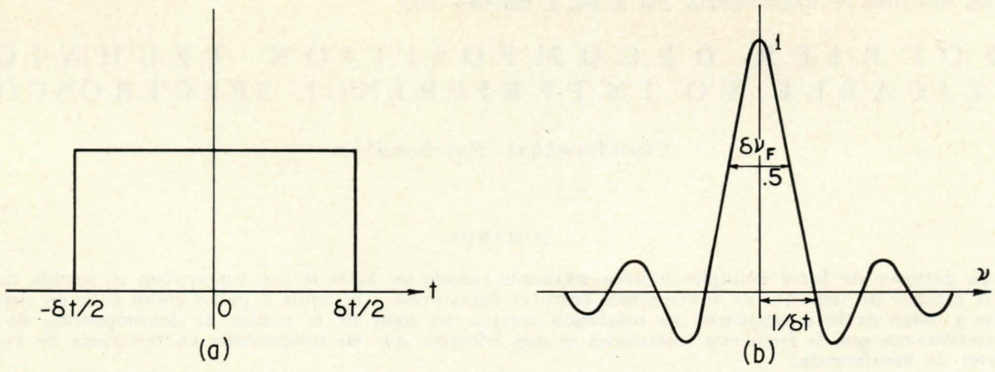

Fig. 1. (a) *An ideal rectangular pulse of light of frequency* ν_0 *of width* δt. (b) *Distribution of the Fourier decomposed monochromatic components of the rectangular pulse.*

where $\delta\nu_G$ is the full width at half maximum. The factor $\cancel{0.9}$ 1.2 appears because the grating resolving power is defined (following the Rayleigh criterion) using the width from the center of the peak to where it goes to zero for the first time; this width is a little ~~larger~~ smaller, $(1/\cancel{0.9}$ 1.2$)$, than the half-width $\delta\nu_G$ we are using. We prefer to use the half-width criterion because of the fact that our instrumental response function does not go to zero for either the FP or the grating. Both the gratings under discussion will give the same instrumental half-width as long as the product mN remains constant (Equation 3). If we accept the Fourier decomposition as a reality, then for each component frequency we should get a grating response of functional form,

$$G(m = \nu\tau_m) = \frac{\sin \pi mN}{\sin \pi m} \tag{4}$$

where $\tau_m = m\lambda/c$, $m\lambda$ being the delay between the consecutive wavefronts in the m-th order. Then the total effective width, $\delta\nu_{FG}$, of the grating pattern produced by a single frequency (ν_0) rectangular pulse, $f(t)$, should be ~~given by~~ obtained from the convolution,

$$G(\nu\tau_m) \oplus F(\nu) \tag{5}$$

Now, suppose we choose a pulse $f(t)$ of width δt such that it contains 10^3 complete optical cycles (implying a pulse of picosecond order in the visible range), that is,

$$\delta t = 10^3/\nu \tag{6}$$

Then, using Equations (2) and (6), the Fourier half-width

$$\delta\nu_F = \frac{\overset{1.2}{\cancel{0.9}}}{\delta t} = \overset{1.2}{\cancel{0.9}} \times 10^{-3}\,\nu, \tag{7}$$

Whereas the grating half-width, from Equation (3) is

$$\delta\nu_G = \overset{1.2}{\cancel{0.9}} \times 10^{-5}\,\nu \tag{8}$$

Thus the width of the monochromatic grating response function $G(\nu\tau_m)$ is two orders of magnitude narrower than the Fourier frequency distribution function $F(\nu)$ for our chosen case. Then the effective width of the grating pattern to the short pulse $f(t)$, in spite of the convolution (relation 5), is approximately that of the Fourier half-width,

$$\delta\nu_{FG} \approx \delta\nu_F \tag{9}$$

Probably, such observations have so far given the impression that the Fourier decomposition technique explains the reality very well.

Let us now look at the very process by which a grating produces its interference (diffraction) pattern. Following Huygens' principle all the slits of a grating produce their own wavelets which in the m-th order direction have a regular delay of $\tau_m = m\lambda/c$ between any consecutive pair. In the case of stationary illumination, all the wavelets could be simultaneously present with the time delay amounting only to a relative phase delay and the grating function of Equation 4 is a reality to any detector. But when the incident radiation constitutes a short pulse, like the one with only 10^3 optical cycles, one can not superpose more than q beams at a time, in the m-th order, where

$$\delta t = q\tau_m = q \frac{m\lambda}{c} = q \frac{m}{\nu} \tag{10}$$

or

$$\nu\delta t = qm = 10^3 \tag{11}$$

(For $m = 1$, $q = 10^3$ and for $m = 10^2$, $q = 10$). Then the maximum resolving power that one can obtain at the most opportune moment with either of the gratings under discussion is

$$\frac{\nu}{\delta\nu_{Gt}/\underset{1.2}{\cancel{0.9}}} = mq = 10^3 \tag{12}$$

or

$$\delta\nu_{Gt} = \overset{1.2}{\cancel{0.9}} \times 10^{-3}\,\nu = \delta\nu_F \tag{13}$$

(where the suffix t emphasizes the fact that such an effect is only transient). Thus one might say that the dynamic resolving power of a grating to a short pulse can not exceed that given by the Fourier decomposition band-width, $\delta\nu_F$. But this is only a momentary phenomena that can be separated only by exotic equipment like a picosecond streak-camera. In reality what happens at the classical spectral plane of the grating is as follows. First, there is uniform energy due to one pulse from one slit for a period of time τ_m; then there is a two-beam pattern for the same period τ_m; then there is a three-beam interference pattern for the same period, and so on up to a $(q - 1)$ beam pattern; then there is a q beam pattern for $(|N - q| + 1)\,\tau_m$ sec and then back to $(q - 1)$, $(q - 2)$, $\cdots$, 2, 1, 0 beam patterns, each existing for a period of τ_m. The total integral exposure is

$$E(m) = 2\tau_m \sum_{p=1}^{q-1} \frac{\sin^2\pi mp}{\sin^2\pi m} + (|N - q| + 1)\,\tau_m \frac{\sin^2\pi mq}{\sin^2\pi m} \tag{14}$$

For an FP the expression is very similar to above, except that the grating function $\sin^2/\sin^2$ is replaced by the appropiate FP function (Roychoudhuri 1975b, 1976)

$$\mathcal{T}_p \frac{1 + F_p \sin^2\pi mp}{1 + F \sin^2\pi m} \tag{15}$$

where $\mathcal{T}_p$, F_p and F are functions of FP mirror transmision and reflection characteristics.

Thus when our model of real physical superposition predicts a spectral curve given by Equation 14, the Fourier decomposition model predicts a convolution relation (5); they are not equivalent. The concept of the limiting dynamic resolving power (Equation 13) takes into account only the last term of Equation 14. The physical meaning of Equation 14, as has already been explained, is that one sums a large number of broad to narrow multiple-beam interference patterns; the resultant will have a wide half-width, wider than the Fourier bandwidth, and an appreciable energy in the tails. Thus our prediction is in direct disagreement with that of Fourier decomposition method. So far, we have not carried out any direct experimental verification, but the existing publications do support us to a limited degree. It is a common observation (see review article by Bradley et al. 1974) that a grating shows a spectral width for picosecond pulses that is wider than the time bandwidth implies. Although the literature explains this effect as exclusively due to self-phase modulation at high energies of short pulses, we believe that a part of it is due to the time evolving interference pattern described by our model of real physical superposition. Further, a long tail in the recorded spectrum of picosecond pulses can be seen in many publications like Von Der Linde et al. (1970), although the point has not been discussed explicitly in the literature.

The same instrumental function of Equation 14 is applicable to spontaneous pulses with random phases for the interference effect between two different pulses averages to zero.

III. *Are the Fourier Monochromatic Components Real?*

From the viewpoint of classical causality it is difficult to accept Fourier decomposition as a physical reality especially for interference spectroscopy. Any signal has a finite velocity if we follow the special theory of relativity; and light in vacuum has the highest velocity. Then an arbitrary shaped but finite size pulse will need a finite time to pass through an FP or a grating. Further, since an FP (a pair of beam splitters) or a grating (a regular array of transparent and opaque lines, or a regularly displaced stack of glass slides or an equivalent) are inert systems without any significant memory (to our knowledge) and are perfectly linear to electromagnetic fields, they can not perform Fourier decomposition. Fourier decomposition requires a complete knowledge regarding the shape and duration of a pulse $f(t)$, for the amplitude and phase of each Fourier component are determined by the totality of information carried by $f(t)$,

$$F(\nu) = \int_{-\infty}^{\infty} f(t) \exp(2\pi i \nu t) dt \tag{16}$$

So, only systems with a special type of memory can carry out such a detailed integration. It is the property of the polychromatic field modified in the presence of an FP or a grating and not due to any special property of these instruments that the energy corresponding to different optical frequencies is separated. Naturally, the response of these instruments regarding their capability to separate different frequencies will be different for steady state and transient illuminations. Further, we do know that the mathematical decomposition into sinusoidal components is not unique. It can be decomposed into other functional forms also. Then which decomposition is physical and which is metaphysical? For example, Van der Pol (1953, Van Name, 1954) has shown that a saw-tooth wave can be decomposed into a set of either orthogonal square waves or sinusoidal waves.

But let us assume, for the sake of argument, that the Fourier components exist in reality. Then, is there a way to demonstrate it with a linear system? We know that when two collimated beams of the same frequency and of steady relative phase interfere, they produce three dimensional planar fringes of cosine energy distributions, but only in the region of real physical superposition. These fringes are stationary in space. But if one of the beams has a different effective frequency but otherwise steady phase relation with the other, the planar fringes move with a velocity proportional to the frequency difference (beat) in a perpendicular direction keeping themselves parallel to each other. If one has a series of beams, instead of two, one can see sharp multiple-beam fringes, instead of the wide cosine type, which are stationary in space for single frequency and move steadily in the usual orthogonal direction if their effective frequencies differ from each other by a regularly varying factor. This last experiment can be simulated easily using a Fabry-Perot illuminated by a narrow laser beam incident at an angle at one end of the mirror pair (Roychoudhuri 1975c). Then the FP produces a series of multiply reflected beams spatially separate but parallel to each other (Figure 2). A lens can

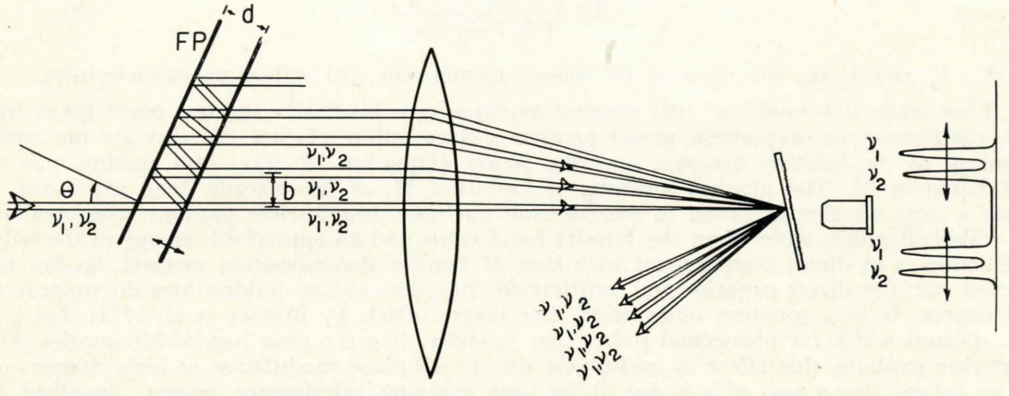

Fig. 2. *A narrow laser beam incident on a Fabry-Perot at an angle θ produces many parallel reflected beams. They can be superposed by a lens to produce multiple-beam interference fringes. These fringes move laterally when one of the mirrors is given a velocity.*

focus this series of beams to produce sharp fringes. Now, if one of the mirrors is given a steady velocity v, the beam which comes out after n reflections from the moving mirror has a Doppler-shifted frequency

$$\nu_n = \nu_0 \left(1 + \frac{2v \cos \theta}{c}\right)^n \tag{17}$$

where θ is the angle of incidence of the laser beam. The effect of such a Doppler shift is to make the sharp planar fringes move in a direction perpendicular to themselves with a velocity proportional to v.

With this background, we can argue that if the Fourier components of a pulse were real, then after passing through a plane grating each of the component frequencies should propagate in a different direction,

$$b \sin \theta_n = m\lambda_n = mc/\nu_n \tag{18}$$

where b is the grating constant and m is the order of diffraction. If all these diffracted frequencies are superposed by an imaging lens (Figure 3), then they should produce the usual sharp planar fringes

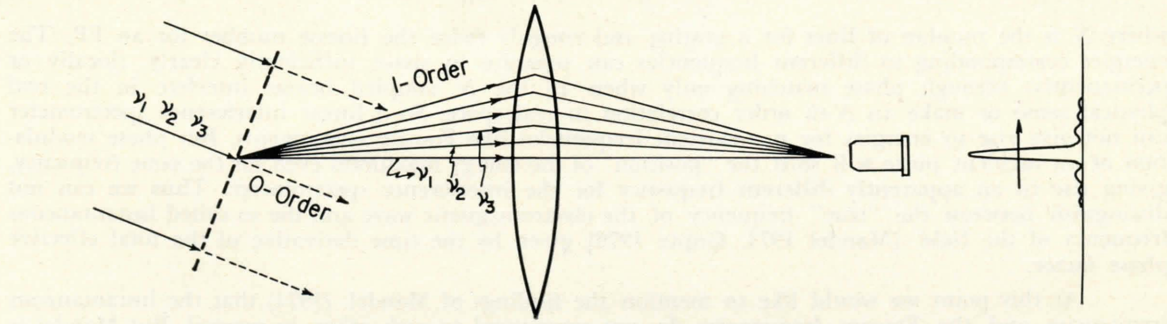

Fig. 3. *If the Fourier monochromatic components exist in reality they should be diffracted at various angles by a grating. If they are superposed again, the beat signal should show up as a lateral movement of the interference fringes.*

moving in a direction perpendicular to their own plane. The effect should be observable with proper arrangements if the Fourier components are real for they must have a steady phase relation to each other by Fourier transform relation. We hope to report in the near future some experiments along these lines.

Let us come back to the basic concepts of interference spectroscopy with FPs and gratings. Both of them sample the incident beam to produce a train of beams with a regular path delay (Figure 4)

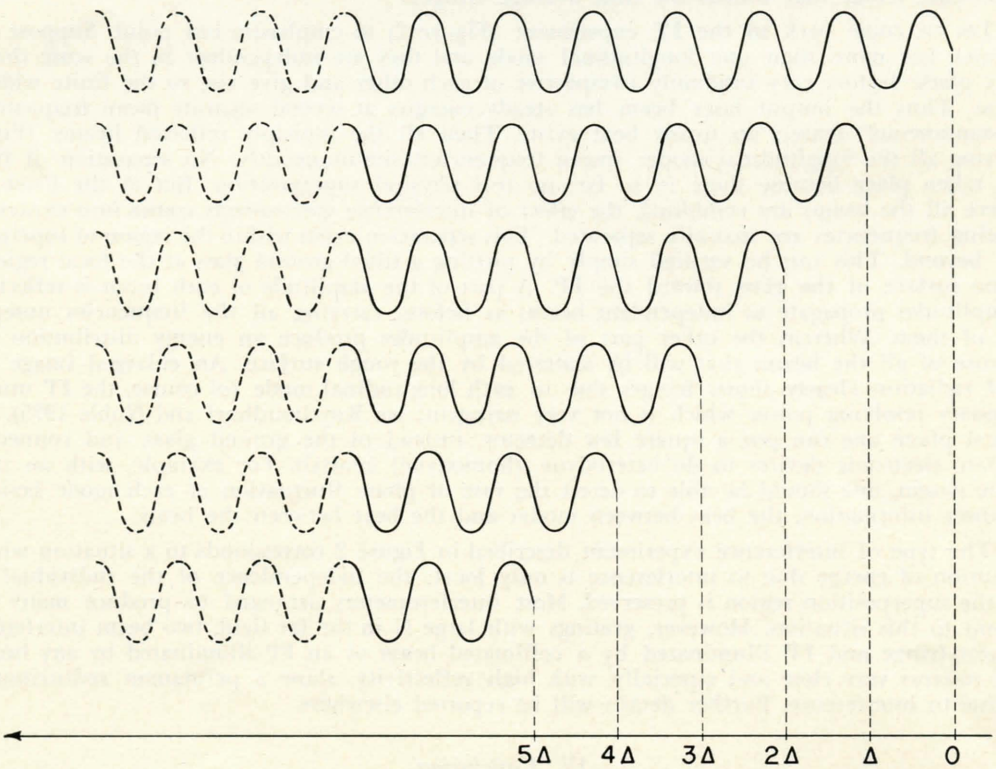

Fig. 4. *Interference spectrometers like Fabry-Perots and gratings replicate the incident beam into a train of regularly delayed (say* Δ*) beams. Real physical interference of this train of beams is the reason behind separation of energies due to different (really) existing frequencies. The free spectral range is* $c/\Delta = \nu/m$ *and the resolving power is* $N\Delta/\lambda = mN$*, where* m *is the order of interference and* N *is the number of interfering beams.*

either through amplitude division (FP) or wave front division (grating.) If the path delay between any pair of consecutive beams is Δ, the free spectral range,

$$\delta\nu_{fsr} = \frac{c}{\Delta} = \frac{c}{m\lambda} = \frac{\nu}{m} = \frac{1}{\tau_m} \tag{19}$$

and the resolving power,

$$\mathcal{R} = \frac{N\Delta}{\lambda} = mN = \frac{\nu}{\delta\nu} \tag{20}$$

where N is the number of lines for a grating and roughly twice the finesse number for an FP. The energies corresponding to different frequencies can separate in space sufficiently clearly (locally or permanently) through phase matching only when all the N sampled beams interfere in the real physical sense or make an N-th order correlation in real space. So a linear interference spectrometer can not give rise to energies for non-existent frequencies like Fourier components. But phase modulation of an incident pulse will shift the "position" of the energy maximum even for the same frequency, giving rise to an apparently different frequency for the interference spectrometer. Thus we can not distinguish between the "true" frequency of the electromagnetic wave and the so called instantaneous frequency of the field (Mandel 1974, Gupta 1976) given by the time derivative of the total effective phase factor.

At this point we would like to mention the findings of Mandel (1974) that the instantaneous frequencies and the Fourier frequencies do not correspond to each other, in general. But Mandel is of the opinion that Fourier component frequencies are relevant for interference spectroscopy and instantaneous frequencies for heterodyne spectroscopy. We believe that it is always the instantaneous frequency that is relevant in any interference experiment, although we can measure only the time-average of the instantaneous frequency due to the lack of an instantaneous detector. A resultant electromagnetic field, produced by whichever kind of interference, can have no knowledge as to whether the square-law detector, placed at the region of superposition, is connected to measure the low frequency beat signals (heterodyne spectroscopy) or to record only the time average spatial energy distribution (so called interference spectroscopy; in the latter case the beat signal is equivalent to lateral movement of interference fringe that widens the time average fringe).

Let us come back to the FP experiment (Figure 2) to emphasize our point. Suppose the incident laser has more than one longitudinal mode and they are independent in the sense that their arbitrary phase factors vary randomly irrespective of each other and give rise to the finite width of a laser line. Thus the output laser beam has steady energies at several separate mean frequencies (in the "instantaneous" sense); no steady beat exists. Then all the multiply reflected beams (Figure 2) are carrying all the longitudinal modes (mean frequencies) simultaneously. No separation of frequencies has taken place because there is, so far, no real physical superposition. But at the focus of the lens where all the beams are combined, the effect of interference spectroscopy comes into existence and the existing frequencies are spatially separated. This separation exists within the region of superposition and not beyond. This can be verified simply by putting a tilted ground glass at the focal region with the plane surface of the glass toward the FP. A part of the amplitude of each beam is reflected and these amplitudes propagate as independent beams as before, carrying all the frequencies unseparated in each of them. Whereas the other part of the amplitudes produce an energy distribution due to interference of all the beams that will be scattered by the rough surface. An enlarged image of this scattered radiation clearly shows fringes due to each longitudinal mode (of course, the FP must have the necessary resolving power which is not very stringent; see Roychoudhuri and Noble 1975). In this same focal plane one can put a square law detector, instead of the ground glass, and connect it to appropriate electronic devices to do heterodyne (homodyne) analysis. For example, with an ultrafast electronic system, one should be able to detect the rate of phase fluctuation of each mode besides the much slower information, the beat between modes and the beat between the beats.

The type of interference experiment described in Figure 2 corresponds to a situation where the redistribution of energy due to interference is only local; the independence of the individual beams beyond the superposition region is preserved. Most interferometers arranged to produce many fringes correspond to this situation. However, gratings with large N in the far field, two beam interferometers set for zero-fringe and FP illuminated by a collimated beam or an FP illuminated by any beam but with the mirrors very close and especially, with high reflectivity, show a permanent redistribution of energy due to interference. Further details will be reported elsewhere.

IV. *Conclusion*

We have shown from the first principle that the grating and Fabry-Perot patterns due to short pulses of light have width and tail energy greater than that predicted by the Fourier decomposition technique. We have offered as indirect support some of the experimental findings by other people. Then we have described an experiment which might be able to test whether the Fourier monochromatic components physically exist. But from the very structure of Fourier decomposition integral and the principle of causality, we believe that corresponding monochromatic components can not be experienced by linear systems like Fabry-Perots and gratings. In the process of these arguments, we found that the so called instantaneous frequency (the time derivative of the total phase factor), rather than the Fourier frequency, corresponds more closely to the interference experiments.

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